DEC 8 1930

JOURNAL
of the
Optical Society of America

VOLUME 20
No. 12

DECEMBER
1930

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Published monthly by the OPTICAL SOCIETY OF AMERICA
Office of Publication
GEORGE BANTA PUBLISHING COMPANY, 450 Ahnaip St., Menasha, Wis.

JOURNAL of the Optical Society of America

Volume 20 **Number 12**

Published monthly by the Optical Society of America. Devoted to original papers on experimental, theoretical, and applied optics. Manuscripts for publication are invited, whether authors are members of the Optical Society or not.

Annual subscription $4.00 dating from January of current year; or $5.00 in combination with THE REVIEW OF SCIENTIFIC INSTRUMENTS. Single copies, $0.60.

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Volume 20— (1930-), one volume per year, per volume..... $ 4.00

Address publishers, "George Banta Publishing Company, 450-454 Ahnaip Street, Menasha, Wisconsin," or "F. K. Richtmyer, Managing Editor, Cornell University, Ithaca, New York," for all correspondence regarding subscriptions, back numbers, changes of address, advertising rates and accounts, and missing numbers.

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Entered as second-class matter May 31, 1921, at the post office at Menasha, Wisconsin, under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412, P. L. and R., authorized June 19, 1925.

Journal
of the
Optical Society of America

Vol. 20 DECEMBER, 1930 No. 12

THE MIXTURE DATA EMBODIED IN THE TENTATIVE CURVES OF HECHT'S THEORY OF VISION*

By Deane B. Judd

[Washington, D. C. Received September 15, 1930]

In a recent number of this journal,† Dr. Selig Hecht tentatively proposed a set of three curves to represent the spectral characteristics of the three types of cones in his extension of the Young theory of vision. These curves were built up according to rules of computation discussed in his paper to represent both Laurens' data on sensibility to wavelength difference at unit colorimetric purity and the data of Priest and Brickwedde on sensibility to colorimetric-purity difference near zero purity. About these curves Dr. Hecht has constructed a theoretical and speculative structure which has attracted considerable interest. The plausibility of the three-components hypothesis in this particular modification is forcibly argued and the whole article, which is meant to be suggestive and stimulating rather than rigorous and final, fairly bristles with new viewpoints and should result in a considerable and valuable impetus to achievement in visual theory.

Dr. Hecht states that the curves which he has tentatively proposed constitute an adequate description of the facts of "color-mixture." The evidence which is offered in support of this statement is of two kinds: (1) a comparison of the curves with a linear, homogeneous transformation of the O. S. A. "excitation" curves, and (2) a check against Sinden's data on complementary stimuli. The first comparison is given in his Fig. 8 and it is apparent at a glance that his proposed theoreti-

* Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

† Selig Hecht, The Development of Thomas Young's Theory of Color Vision, J.O.S.A., *20*, 231–270; 1930.

cal curves may be closely duplicated by a homogeneous, linear transformation of the O. S. A. "excitation" curves, which closely represent the "color-mixture" relations of the normal visual mechanism. If it could be shown that Dr. Hecht's curves duplicate *exactly* a linear, homogeneous transformation of the O. S. A. "excitation" curves his conclusion would be indisputable. Whether the conclusion is correct, however, can not be judged from his comparison because we can not be sure that the apparently unimportant discrepancies are really negligible. Indeed, there seems to be ground for believing the contrary. An outstanding characteristic of the proposed set of curves is that the three are nearly identical;[1] that is, the whole of the chromatic aspect of vision is ascribed to the relatively small differences between the three curves. Is it not reasonable, then, to suspect that the apparently negligible discrepancies which he found might be serious?

It is the purpose of the present paper (1) to present a more adequate comparison of Dr. Hecht's tentative theoretical curves with the O. S. A. "excitation" curves, and (2) to discover whether his method of comparison has led into error.

An obvious way to make a better comparison is to adopt such a transformation that the three curves of each set are quite distinct. We propose to accomplish this by referring Dr. Hecht's curves to the primary colors whose distributions throughout the spectrum are given by the O. S. A. "excitation" curves; that is, we shall transform the theoretical curves and make the comparison with the O. S. A. "excitation" curves directly. If the theoretical curves embody essentially the mixture data of the O. S. A. "excitation" curves, as Dr. Hecht's comparison led him to conclude, this transformation will also agree essentially with the O. S. A. "excitation" curves.

The method of making these transformations has long been known. If R, G, and V represent Dr. Hecht's curves (see Table 1, columns 2, 3, and 4 which were copied from Dr. Hecht's Table 1), then r, g, and v, the curves of the required transformation, are defined:

$$\left.\begin{aligned} r &\equiv K_1R + K_2G + K_3V \\ g &\equiv K_4R + K_5G + K_6V \\ v &\equiv K_7R + K_8G + K_9V \end{aligned}\right\} \qquad (1)$$

where the constants, K_1 to K_9, are chosen appropriately. The methods of choosing the constants for various purposes have been discussed

[1] Hecht, l. c., pp. 251, 252; see his Table 1 and Fig. 7.

TABLE 1. *Hecht's tentative theoretical curves, the O. S. A. "excitation" curves, and Hecht's curves transformed so as to refer to the O. S. A. primaries.*

1	2	3	4	5	6	7	8	9	10
λ in $m\mu$	R	G	V	r_0	g_0	v_0	r	g	v
380						442			
390						1274			
400						2717			
410	.1	1.4	2.5			5045	−1	−1	1037
420	.5	3.7	5.9		13	7679	−40	35	2205
430	2.0	6.1	9.9		41	12186	95	19	3660
440	4.4	9.3	15.3		100	14351	332	−18	5611
450	7.7	12.8	21.5		240	14027	701	−92	7903
460	12.4	17.6	28.1	15	595	12958	1044	−87	9642
470	21.8	29.5	38.7	64	1313	11098	1064	319	9941
480	35.7	48.8	53.5	231	2040	7767	700	1149	8800
490	58.6	78.5	75.0	693	2897	3703	202	2463	5982
500	97.8	124.6	113.6	1430	4546	2112	383	4107	5105
510	160.1	175.4	144.6	2644	6944	1517	1810	6107	−5946
520	211.4	229.9	196.7	4125	9160	1076	3254	7548	−2696
530	262.9	284.3	248.9	5479	10357	765	4750	8967	664
540	303.7	325.1	289.2	6681	10958	517	6252	9908	3834
550	329.8	348.4	313.9	7604	11134	322	7613	10278	6630
560	331.2	346.2	321.6	8325	10475	196	8927	9560	13351
570	317.2	325.6	310.2	8989	9336	124	9841	8397	17703
580	301.5	299.4	278.1	9210	7456	71	9564	7889	10212
590	267.7	256.3	234.0	9388	5269	35	8971	6749	4938
600	229.6	212.7	190.7	8841	3447	17	8020	5616	900
610	181.7	161.4	146.9	7900	1960		7213	3945	1631
620	139.7	119.4	103.0	6291	1004		5436	3117	−3394
630	99.0	81.7	69.4	4690	484		4033	2110	−3406
640	67.2	52.4	44.4	3161	164		3030	1265	−2589
650	43.2	30.3	26.5	1871	48		2364	574	−1273
660	27.6	16.6	15.8	1057	16		1901	141	−106
670	19.1	10.0	9.9	606			1484	0	1
680	12.2	5.0	4.8	326			1053	−58	−341
690				187					
700				105					
710				54					
720				33					
Totals	3545.8	3612.4	3312.6	100000	99997	99999	99996	100007	99999

before[2] and, although there is nothing essentially new in it, I give in detail my derivation of the constants needed in this case because often a specific problem solved constitutes a more helpful description of a method than the most elegant general treatment.

The constants, K_1 to K_9, required are those which express the mixture data embodied in Dr. Hecht's theoretical curves, R, G, and V, in terms of the primaries of the O. S. A. "excitation" curves; but this alone is not sufficient to ensure comparability. What we should like to do is to find a transformation of the theoretical curves which would not only embody the primaries of the O. S. A. "excitation" curves, but which would also refer to the same "neutral" standard[3] and which would in addition be expressed in the same scale numerals. Such a transformation would be directly comparable to the O. S. A. "excitation" curves.

The colors corresponding to homogeneous radiant energy of wave length 410 and 670mμ respectively constitute two of the three primary colors of the O. S. A. "excitation" curves. From Table 1 we see that $R=0.1$, $G=1.4$, and $V=2.5$ for 410mμ. Since the desired violet primary in the required transformation must correspond to this wave length it is plain that we must set: $r=0$, $g=0$, for 410mμ. Hence, from relation (1) we have:

$$0.1K_1 + 1.4K_2 + 2.5K_3 = 0 \tag{2}$$

$$0.1K_4 + 1.4K_5 + 2.5K_6 = 0. \tag{3}$$

From Table 1 we also find that, for 670mμ, $R=19.1$, $G=10.0$, and $V=9.9$, and, since we wish to make $g=b=0$ for this wave length, we have:

$$19.1K_4 + 10.0K_5 + 9.9K_6 = 0 \tag{4}$$

$$19.1K_7 + 10.0K_8 + 9.9K_9 = 0 \tag{5}$$

The third primary color of the O. S. A. "excitation" curves corresponds (for the visual mechanism in the chromatically neutral state)[4] to a color stimulus that is not physically realizable; that is, the stimulus

[2] See, for example, J. Guild, *The Transformation of Trichromatic Mixture Data: Algebraic Method*, Trans. Opt. Soc., *26*, 95–108; 1924–25.

[3] Dr. Hecht's curves refer to the spectrum of Abbot-Priest sunlight; we take, therefore, the O. S. A. "excitation" curves for this spectrum. It is natural and convenient to make Abbot-Priest sun the "neutral" standard also; that is, we represent the color of Abbot-Priest sunlight at the center of the color triangle both for the O. S. A. "excitation" curves and for the transformed theoretical curves.

[4] For definition in explicit terms of the nearly self-explanatory phrase "chromatically neutral state" see D. B. Judd, *Reduction of Data on Mixture of Color Stimuli*, B. S. Jour. Research, *4*, 541; 1930; particularly footnote 61.

has a dominant wave length of 503.6mμ, but its colorimetric purity is greater than unity (which is the colorimetric purity of the spectrum), hence we say that the primary green of the O. S. A. "excitation" curves is more saturated than spectrum green. On this account we can not, as was done for the primary red and primary violet, deduce two relations which will directly ensure identity of the green primary process embodied in the transformation, r, g, v, with the green process embodied in the O. S. A. "excitation" curves. If, however, the functions R, G, V, embody the mixture data of the O. S. A. "excitation" curves we may indirectly ensure the desired green primary process by writing a pair of relations ensuring identity in trilinear coordinates between r, g, and v, and the O. S. A. "excitation" curves for any one of the remaining 26 points of the spectrum specified by Dr. Hecht (that is, any point specified in Table 1, columns 2, 3 and 4 except the two, 410 and 670mμ, which have already been used) provided we also make the areas beneath the curves of r, g, and v equal.[5] We choose to ensure that r, g, and v be identical in trilinear coordinates with the O. S. A. "excitation" curves at 530mμ, taking this point from the 26 candidates because it promises to avoid easily errors of rejection in computation which would be inconvenient to avoid if the third point had been chosen near either 410 or 670mμ. If r_0, g_0, and v_0 represent the O. S. A. "excitation" curves referred to Abbot-Priest sunlight as the "neutral" standard,[6] the conditions of equality in trilinear coordinates are:

$$r/(r + g + v) = r_0/(r_0 + g_0 + v_0)$$
$$g/(r + g + v) = g_0/(r_0 + g_0 + v_0).$$

We could substitute for either of these conditions:

$$v/(r + g + v) = v_0/(r_0 + g_0 + v_0)$$

but it is evident that only two of the three conditions are necessary since the third can be derived from the others. For 530mμ we see from relation (1) and Table 1 that:

[5] The argument is that two sets of distribution curves referring to the same "neutral" standard (ensured by the condition imposed on the areas beneath the curves) having two of the three primary colors identical will be everywhere identical in trilinear coordinates provided (1) they embody the same mixture data, and provided (2) they are identical for a single other point. Since the two sets will be everywhere identical in trilinear coordinates, they will, in particular, be identical as to green primary.

[6] These are the symbols used by Dr. Hecht, l. c., p. 254. The values of r_0, g_0, and v_0 are given in our Table 1, columns 5, 6, and 7. They have also appeared in a previous paper (D. B. Judd, *Reduction of Data on Mixture of Color Stimuli*, B. S. Jour. Research, *4*, 525; 1930).

$$\frac{262.9K_1 + 284.3K_2 + 248.9K_3}{262.9(K_1 + K_4 + K_7) + 284.3(K_2 + K_5 + K_8) + 248.9(K_3 + K_6 + K_9)} = \frac{5479}{16601} \tag{6}$$

$$\frac{262.9K_4 + 284.3K_5 + 248.9K_6}{262.9(K_1 + K_4 + K_7) + 284.3(K_2 + K_5 + K_8) + 248.9(K_3 + K_6 + K_9)} = \frac{10357}{16601}. \tag{7}$$

To these conditions we have to add two which ensure that the two sets of curves refer to the same "neutral" standard, that is, we require that the areas beneath the curves of r, g, and v be equal. Finally, the areas beneath the curves of r, g, and v are made not only equal to each other but also equal to the areas beneath the O. S. A. "excitation" curves. This added restriction makes three conditions instead of two; it ensures that r, g, and v have the same scale numerals as r_0, g_0, and v_0 and completes the list of conditions to be satisfied in order that the two sets of curves be directly comparable:

$$3545.8K_1 + 3612.4K_2 + 3312.6K_3 = 100{,}000 \tag{8}$$
$$3545.8K_4 + 3612.4K_5 + 3312.6K_6 = 100{,}000 \tag{9}$$
$$3545.8K_7 + 3612.4K_8 + 3312.6K_9 = 100{,}000. \tag{10}$$

From Table 1 it may be seen that the numbers 3545.8, 3612.4 and 3312.6 indicate respectively the totals of columns 2, 3 and 4, while 100,000 indicates closely the totals of columns 5, 6, and 7.

Relations (2) to (10) constitute a set of nine simultaneous equations involving nine unknowns, K_1 to K_9; by solving these nine equations we find the constants of relation(1) required to transform Dr. Hecht's tentative theoretical curves into a set of curves directly comparable with the O. S. A. "excitation" curves. The equations of transformation thus found are:

$$\left.\begin{aligned} r &\equiv 122.534R - 181.096G + 96.513V \\ g &\equiv -25.234R + 105.919G - 58.305V \\ v &\equiv -87.196R - 555.934G + 729.769V \end{aligned}\right\} \tag{11}$$

Columns 8, 9, and 10 of Table 1 give r, g, and v so calculated,[7] and Figs. 1 and 2 serve to compare r, g, and v with the O. S. A. "excitation" curves, r_0, g_0, and v_0.

[7] Note that the algebraic sum of the numbers specifying r is 99,996, the corresponding sum for g is 100,007, and that for v is 99,999. We may conclude from this that relations (8), (9), and (10) have been satisfied to within one part in 10,000.

Examination of Fig. 1 shows that r and g bear considerable resemblance to r_0 and g_0, respectively, and perhaps one might think that no serious error would result from the substitution of r and g for r_0 and g_0. Fig. 2, however, warns in an unmistakable way against the safety of substituting v for v_0. Now since r_0, g_0, and v_0 are the O. S. A. "excitation" curves which are known to represent closely the "color-mixture" relations of the average normal eye, and since v fails by a wide margin to agree with v_0, we are justified in suspecting that r, g, and v fail in important respects to represent those relations. Furthermore, we are justified

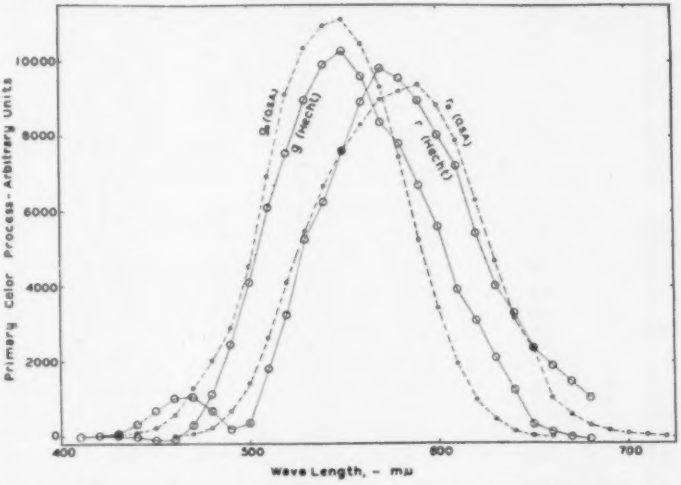

FIG. 1. *Comparison of a linear homogeneous transformation of Hecht's tentative theoretical curves with the O. S. A. "excitation" curves,—red and green.*

This transformation embodies the primaries of the O. S. A. "excitation" curves and the same "neutral" standard.

in entertaining a similar suspicion with respect to the theoretical curves themselves, in spite of plausible evidence adduced by Dr. Hecht to the contrary, because those curves, R, G, V, embody the same mixture relations as r, g, and v.[8] It remains to demonstrate the extent of the discrepancy.

If it had been proven by adequate statistical investigation[9] that the O. S. A. "excitation" curves represented the exact average of a large

[8] We know they embody the same mixture relations because r, g, and v from their definitions (see relation (11)) constitute a homogeneous, linear transformation of R, G, and V. See D. B. Judd, *Reduction of Data on Mixture of Color Stimuli*, B. S. Jour. Research, *4*, 515–518; 1930.

[9] As adequate as has been carried out, for example, on the visibility function (see K. S.

number of observers, and if Dr. Hecht's tentative theoretical curves were being put forward as correct and final, we would be justified in expecting to find no discrepancies amounting to more than a least perceptible difference. Since, however, there is some uncertainty in the

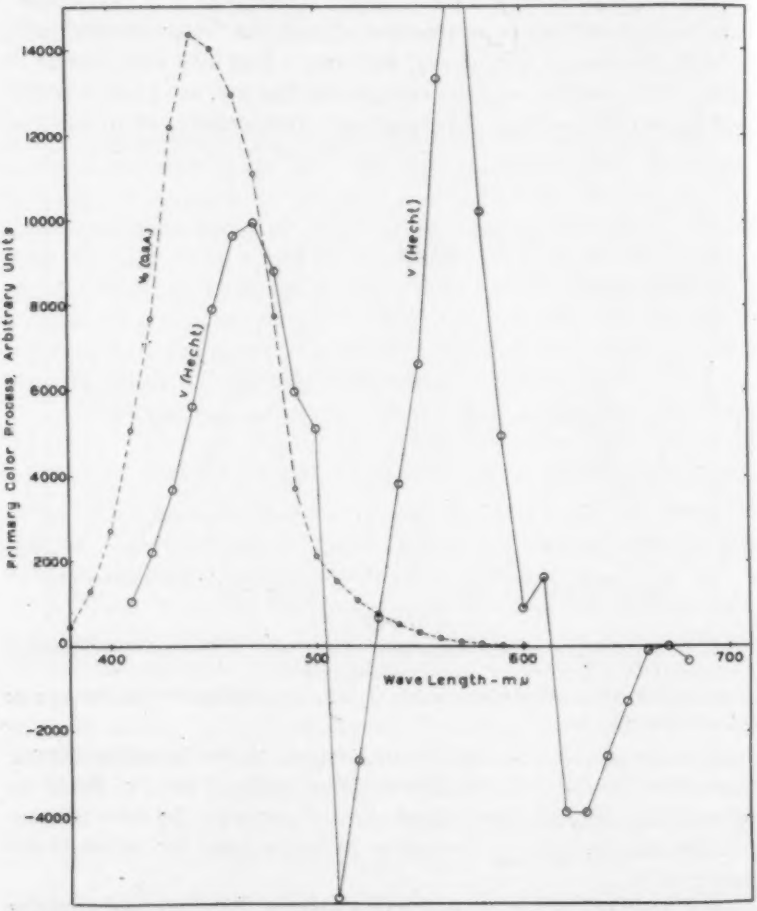

FIG. 2. *Comparison of a linear homogeneous transformation of Hecht's tentative theoretical curves with the O. S. A. "excitation" curves,—violet.*

This transformation embodies the primaries of the O. S. A. "excitation" curves and the same "neutral" standard.

Gibson and E. P. T. Tyndall, *Visibility of Radiant Energy*, B. S. Sci. Paper No. 475, *19*, 131–191; 1923).

TABLE 2. *Trilinear coordinates of the spectrum.*

(A) From the O. S. A. "excitation" curves,

(B) From Hecht's tentative theoretical curves transformed so as to refer to the primary color processes of the O. S. A. "excitation" curves.

	From O. S. A. curves			From Hecht's curves		
1	2	3	4	5	6	7
λ in mμ	r_0/s_0*	g_0/s_0	v_0/s_0	r/s*	g/s	v/s
410	.000	.000	1.000	−.001	−.001	1.002
420	.000	.002	.998	−.018	.016	1.002
430	.000	.003	.997	.025	.005	.970
440	.000	.007	.993	.056	−.003	.947
450	.000	.017	.983	.082	−.001	.928
460	.001	.044	.955	.098	−.008	.910
470	.005	.105	.890	.094	.028	.878
480	.023	.203	.774	.066	.108	.826
490	.095	.397	.508	.023	.285	.692
500	.177	.562	.261	.040	.428	.532
510	.238	.625	.137	.918	3.098	−3.017
520	.287	.638	.075	.401	.931	−.333
530	.330	.624	.046	.330	.624	.046
540	.368	.604	.028	.313	.496	.192
550	.399	.584	.017	.310	.419	.270
560	.438	.551	.011	.280	.300	.419
570	.487	.506	.007	.274	.234	.493
580	.550	.446	.004	.346	.285	.369
590	.639	.359	.002	.434	.327	.239
600	.719	.280	.001	.552	.386	.062
610	.801	.199	.000	.564	.308	.128
620	.862	.138	.000	1.054	.604	−.658
630	.906	.094	.000	1.474	.771	−1.244
640	.951	.049	.000	1.776	.742	−1.518
650	.975	.025	.000	1.420	.345	−.765
660	.985	.015	.000	.982	.073	−.055
670	1.000	.000	.000	.999	.000	.001
680	1.000	.000	.000	1.610	−.089	−.521

* $s_0 = r_0 + g_0 + v_0$; $s = r + g + v$.

O. S. A. "excitation" curves due to the considerable individual variation and since the theoretical curves are intended only as a rough indication of the final curves, differences of as much as 5 times that just perceptible might perhaps be expected. It has been discovered empiri-

cally for this particular set of primary colors that a difference in r-coordinate or a difference in g-coordinate of 0.01 corresponds to one least perceptible difference under usual observing conditions[10] to within a factor of 2[11]. On this account a convenient way of testing the magnitude of the discrepancies between the two sets of curves is to compare the r and g trilinear coordinates of the spectrum colors from the one set of curves with those from the other. Table 2 shows the trilinear coordinates of the spectrum both for the O. S. A. "excitation" curves (columns 2, 3, and 4) and for Dr. Hecht's tentative theoretical curves referred to the same primary colors and "neutral" standard (columns 5, 6, and 7). It will be seen in the first place that the trilinear coordinates of the spectrum for 410, 530, and 670mμ are the same for both sets of curves except for errors ascribable to rejection in computation. This serves to demonstrate that relations (2) to (7) have been satisfied.[12] Aside from these three coincidences which follow from the conditions imposed, the agreement is quite poor; in many cases discrepancies occur which are much greater than 0.10 (known to be more than 5 least perceptible differences) as, for example, at 510mμ where the values of the g-coordinate differ by more than 2.40. This difference is more than 24 times that which could reasonably be expected between two sets of curves each of which could be said to represent normal mixture data even roughly. Since Dr. Hecht was led to conclude that these sets of data are essentially identical, and since we have demonstrated that serious discrepancies exist between them, it is plain that the method of comparison used by Dr. Hecht is quite inadequate.

It might be remarked that the method used by Dr. Hecht is generally as sound as the one used in the present paper. Only when, as is the case with Dr. Hecht's curves, the three curves are nearly identical can a real discrepancy exist undiscovered[13].

[10] Usual observing conditions may be taken to mean observation by one eye of a field of 2 or 3° with a dark surrounding field.

[11] D. B. Judd, *Saturation of Colors Determined from the Visual Response Functions*, J.O.S.A. & R.S.I., *16*, 115; 1928. This reference cites an abstract; the complete paper is in preparation.

[12] It has already been demonstrated that relations (8), (9), and (10) are satisfied, see footnote *7*, p. 652; hence we may be sure that r, g, and v constitute the particular linear, homogeneous transformation of R, G, and V, which we wished to obtain.

[13] A way to test directly whether apparently negligible discrepancies in R, G, or B are really important for chromatic vision has been suggested by Dr. L. B. Tuckerman. We have only to make the comparison for each curve expressed as a difference from the mean of all three curves. Since the whole chromatic aspect of vision is expressed by the variation of these differences with wave length, it will be immediately apparent whether a given discrepancy is important or negligible in chromatic vision.

Consider next the check on the tentative theoretical curves carried out by Dr. Hecht against the data on spectral complementaries by Sinden.[14] This check apparently resulted in a corroboration of the pre-

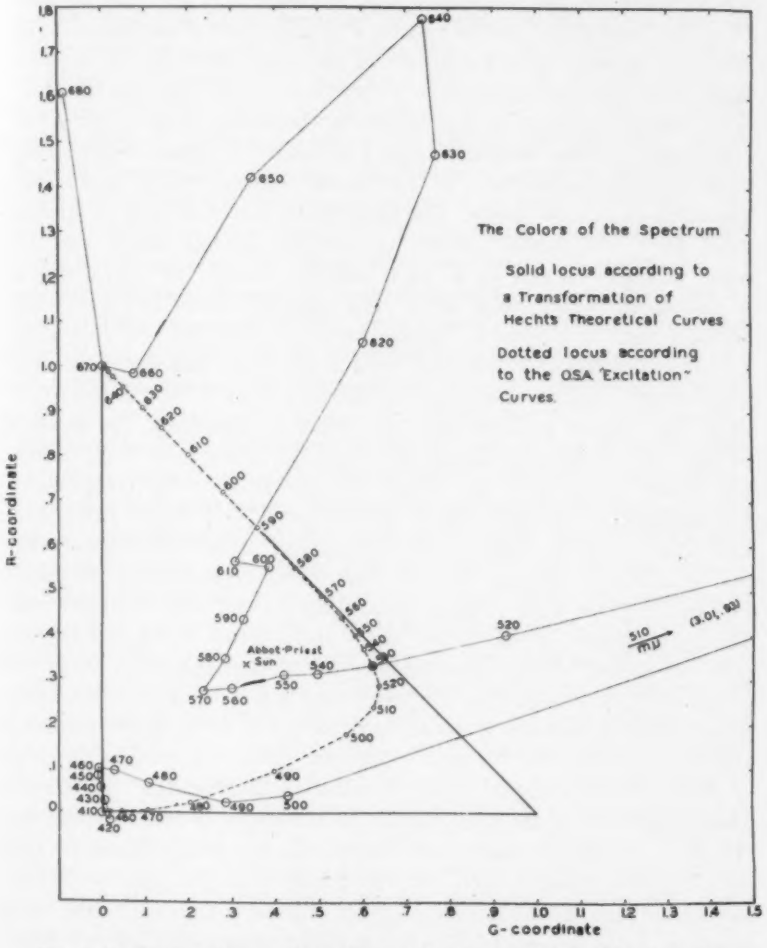
The Colors of the Spectrum
Solid locus according to a Transformation of Hechts Theoretical Curves
Dotted locus according to the OSA "Excitation" Curves.
Abbot-Priest x Sun
R-coordinate
G-coordinate

FIG. 3. *The spectrum locus on the r–g diagram according to the O. S. A. "excitation" curves compared to the spectrum locus according to Hecht's tentative theoretical curves referred to the primaries and "neutral" standard of the O. S. A. "excitation" curves.*

It may be seen that the mixture data embodied in Hecht's curves are markedly different from those embodied in the O. S. A. "excitation" curves.

[14] Hecht, l. c., 264; see his Table 2.

vious conclusion which has just been shown to be in error. Here, as before, the check is apparently satisfactory, but is really inconclusive because there is no indication of what size discrepancies are permissible. To substantiate this statement it is convenient to plot for both the O. S. A. "excitation" curves and for r, g, and v, the r-coordinate of the spectrum against the g-coordinate of the spectrum (see Fig. 3). The large circles plotted from columns 5 and 6 of Table 2 indicate the spectrum locus according to the tentative theoretical curves; the small circles plotted from columns 2 and 3 of Table 2 indicate the spectrum locus according to the O. S. A. "excitation" curves, that is, very closely according to its true course. The large circles are connected by straight solid lines, the small circles by straight dotted lines, and the color triangle (which is a right isoceles triangle on this diagram) is shown in heavy solid lines. The color of Abbot-Priest sunlight for both sets of curves is represented at r-coordinate = g-coordinate $= 1/3$, and the point is indicated by a cross. This diagram constitutes a Newton's mixture diagram, so called because it has the property that the points representing the colors produced by the mixture of any two component stimuli lie along the straight line connecting the points representing the colors corresponding to the components. For example, we see from Fig. 3 (dotted locus) that Abbot-Priest sunlight may be closely matched by a mixture of light of wave length 480 mμ with that of 580mμ because the points representing the colors evoked by those three stimuli lie closely on the same straight line. Merely to glance at the dotted locus, then, shows many of the properties of normal vision; for example, the dotted spectrum locus together with the straight line (g-coordinate equal to zero) connecting its extremes constitutes a closed curve which is at no point concave, and which encloses the "neutral" point (the point representing the color of Abbot-Priest sunlight, which is taken as the "neutral" standard). These properties of the spectrum locus correspond to the experimental fact that, chromatic adaptation being constant, the mixture of two stimuli never evokes a more saturated color than the spectrum color having the same dominant wave length. Likewise a glance at the solid spectrum locus which refers to the tentative theoretical curves[15] demonstrates graphically the discrepancies which were previously shown

[15] Note that the solid locus is coincident with the dotted locus for 410, 530, and 670 mμ. This coincidence is necessitated by relations (2) to (7). That is, we have made the two sets coincident for these three points and for the "neutral" point (relations (8), (9), and (10)) in order to make them comparable.

in Table 2. It is seen that this locus is highly irregular with many concavities. In particular the straight line joining the extremes of the locus cuts it twice (about 440 and 460mμ). The interpretation of this particular property is that, in the mechanism tentatively proposed to represent vision, light of 680mμ added to light of 410mμ would produce a match first for approximately 440mμ then, as more light of 680mμ was added, would produce a match for 460 mμ. In normal vision, of course, the addition of light of 680mμ (red) to light of 410 mμ (reddish-blue) instead of producing a match for light of 460 mμ (blue) rather serves to increase the difference. Furthermore, one very marked concavity appears in the solid spectrum locus of such extent that the "neutral" point actually fails to fall inside any of the areas enclosed by the spectrum locus and the straight line joining its extremes. We may interpret this property to mean that light of wave length 560 to 570mμ could be matched within the proposed mechanism by adding Abbot-Priest sunlight to light of wave length shorter than 480mμ. In the normal mechanism it is, of course, impossible to match light of wave length 560 to 570mμ (yellowish-green) with a mixture of sunlight (white) and light of wave length shorter than 480mμ (reddish-blue to blue). Such mixtures appear as pale reddish-blue or pale blue which does not remotely resemble yellowish-green. Many other striking deviations from normality could be mentioned; for example, it is seen that a straight line drawn through the point on the solid locus labeled 580 and that labeled 550 nearly touches the point (indicated by the cross) representing the color of Abbot-Priest sunlight; this is to be interpreted that within the tentatively proposed mechanism light of 550 and light of 580mμ are nearly complementary, while to the normal visual mechanism none of the mixtures of these two stimuli (yellow and yellowish-green) at all resembles white.[16] It is plainly futile in this case to compute "excitation" sums corresponding to quantitative data on spectral complementaries, we may be sure at a glance that no real correspondence can be demonstrated and that any apparent correspondence is misleading.

Summary of Results: A comparison is presented of the mixture relations embodied in Dr. Hecht's tentative theoretical curves with those embodied in the O. S. A. "excitation" curves. It is shown that the two

[16] It should be mentioned that these qualitative deviations from the facts of color mixture could have been discovered by plotting the Newton's mixture diagram in any homogeneous, linear transformation of r, g, and v. In particular, we could have discovered them (though less conveniently) by examining a plot of $R/(R+G+V)$ against $G/(R+G+V)$. But this examination would not indicate the seriousness of any quantitative deviations from the normal (as does Table 2) in a set of curves which agree qualitatively with the normal.

sets differ quite markedly. Since Dr. Hecht's method of comparison led him to believe that these two sets of curves embodied essentially identical mixture relations, it is concluded that the method of comparison is inadequate.

In conclusion it should be stressed that the extension of Young's theory of vision proposed by Dr. Hecht is not disproved by the findings of this paper. We have merely shown that the first tentative set of curves is not acceptable. Dr. Hecht, himself, has pointed out difficulties remaining in his hypothesis which are probably greater obstacles to the ultimate acceptance of his theory than that offered by the demonstration we have given. Perhaps the material of this paper will serve to indicate the difficulty of attempting to build up theoretical response curves from the data of visual sensibility rather than from data arising from mixture of color stimuli as has been done heretofore.

A STUDY OF PIGMENT PRIMARIES AND MIXTURES

By Frank F. Rupert[1]

[Mellon Institute of Industrial Research, Pittsburgh Pa., Received August 13, 1930]

Introduction

The experimental work to be reported in this paper may be considered as an attempt to establish a quantitative basis for the results of mixing of colored paints, especially water colors. Some attention is being given at the present time to this subject. Bruce[2] has studied the effect of the admixture of colored pigments with white and black pigments, with especial reference to brightness and purity. Appel[3] has suggested a method of predicting the spectrophotometric curves of dye mixtures from those of their components.

It will be seen from the results presented in this paper that a satisfactory method of determining exactly the results of mixing of paints has not been discovered, on account of the complexity of mathematical relations and the lack of knowledge of some of the necessary factors. However, in the general investigation discovery of mathematical relations was secondary to two other factors, of which one was assistance in the selection of suitable working primary colors, and the second, and most important, was demonstration that mixing of all colored media is subject to scientific principles.

Such a demonstration is not necessary to scientists who have studied color, but the deficiency in scientific knowledge of color, even among those best trained in the color arts, is appalling, and the author hopes that his results will help educators and artists to some extent to demonstrate the need of scientific knowledge and to bridge the gap between it and practical art. In so doing he is following the lead of Rood,[4] Bradley,[5] and others, who made important advances in the scientific application of color, considering the lack at that time of instruments that are now in general use. More recently, Martin,[6] Munsell[7] and Luckiesh[8] have contributed to this subject.

[1] Industrial Fellow.

[2] H. D Bruce, Bur. Stands. Jour. Res., *1*, 125–150; 1928.

[3] W. D. Appel, (preliminary publication), Papers Presented at Spring Meeting of Textile Research Council, May 24, 1929, at Atlantic City, N. J. pp. 11–15.

[4] O. N. Rood, *Student's Text Book of Color*, 1892.

[5] Milton Bradley, *Water Colors in the Schoolroom*, 1900.

[6] L. C. Martin, *Colour and Methods of Colour Reproduction*, 1923.

[7] A. E. O. Munsell, *A Color Notation, Atlas of Color*.

[8] M. Luckiesh, *Color and Its Applications*, and other books and articles.

SINGLE PIGMENTS

The first necessary step in the study of mixtures was the spectrophotometric examination of single pigments, especially those most used in water color work, consisting largely of aniline lakes. The instrument used was the Keuffel and Esser color analyzer, with the usual magnesia block as the standard reflecting surface.

The materials examined included the following:

1. Dry pigments, mixed with water to a paste, dried, and planed to a level surface.
2. Pans of water color paints as made by commercial methods.
3. Washes from the prepared paints, not made quantitatively.
4. Washes of definite concentration prepared from pigments and binders in approximately the ratios used in practice.

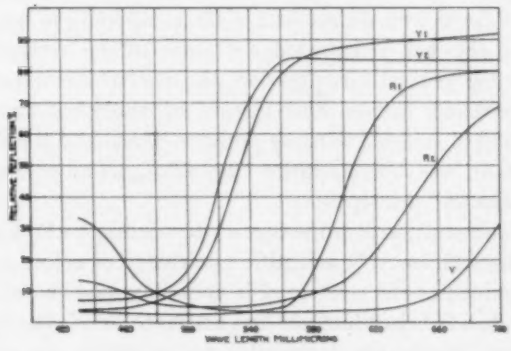

FIG. 1. *Typical dry pigments: R1, Bright scarlet lake; R2, maroon lake; Y1 pale chrome yellow; Y2 brilliant yellow lake; V, violet lake.*

5. A few artists' oil colors brushed on paper in a layer sufficiently thick to be opaque.
6. Colored papers and other miscellaneous materials.

The results obtained were plotted in the usual way, and in most cases dominant wave length, brightness, and purity were calculated according to the methods given in the report on Colorimetry[9]—a publication that was of use in many phases of this investigation.

It would not be of general interest to present any considerable portion of the total number of pigments examined, but a few typical examples

[9] Report of the Colorimetry Committee of the Optical Society of America, 1920–21, L. T. Troland, Chairman, J. O. S. A. & R. S. I. *6*, 527–596; 1922.

will be given, especially since the forms of the curves have an important bearing on the results of mixing.

The curves in Figs. 1 and 2 represent typical dry pigments. Curve *R1*, Fig. 1, is that of the aniline lake "bright scarlet," while is *R2* a "ma-

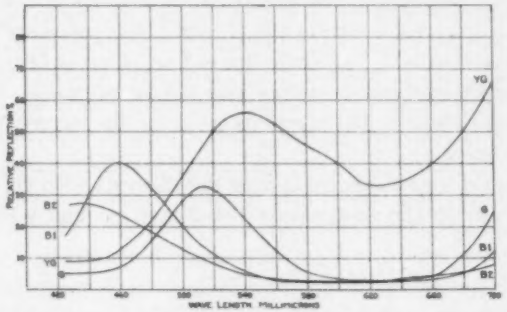

FIG. 2. *Typical dry pigments: B1, peacock blue; B2, blue lake; G, green lake; YG, yellow-green paper.*

roon lake." (Another red, "geranium lake," will be shown in other charts.) Curve *Y1* represents a "pale chrome yellow," and *Y2* a "brilliant yellow lake." *V* is the curve for an intense violet lake. In Fig. 2, the two common types of blue in aniline lakes are represented in curve

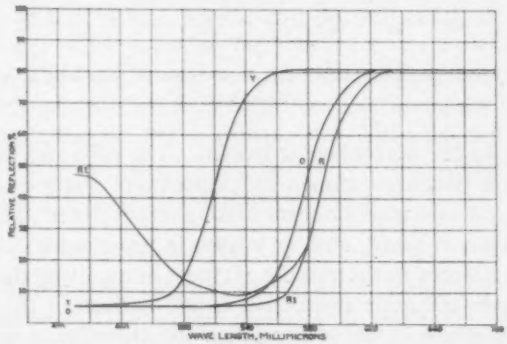

FIG. 3. *Typical water color washes: R1, brilliant scarlet; R2, geranium lake, 4%; R, portion of curves where R1 and R2 agree closely; C, orange lake; Y, naphthol yellow.*

B1, a "peacock blue," and *B2*, a "blue lake." The green lake giving the curve *G* is one of the brightest greens.[10] (Curve *YG* will be considered later.)

[10] For comparison, the curves for certain inorganic pigments given by Luckiesh, Color and Its Applications (1927) p. 298, and reprinted in various works on color, will be of interest.

Curves for typical washes are presented in Fig. 3 and 4. In Fig. 3 small deviations in the red end have been neglected. At wave lengths at which the reflection is high, the curves approach that of the unpainted surface of the paper, which deviates only slightly from 80 percent reflection throughout the spectrum. Curve *R1* is that of a wash of the brilliant scarlet shown in Fig. 1, while *R2* represents a 4 percent wash of "geranium lake," an eosin derivative. Curves *O* and *Y* denote washes from characteristic orange and yellow pigments. In Fig. 4, the washes are those of the pigments used for the curves in Fig. 2, or very similar ones.

These washes were made by mixing the pigment with china clay and dextrin, usually in the proportions 1–1–2, and enough water to bring

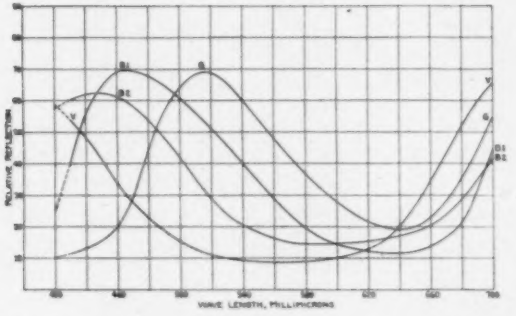

FIG. 4. *Typical water color washes: B1, peacock blue, B2, blue lake; G, green lake; V, violet lake.*

the pigment to the percentage indicated. The clay and dextrin were added because they are common constitutents of water colors and because they produce a more uniform coating on the paper. The wash was applied with a soft brush, such as is used in water color painting, to a heavy paper adapted to the purpose. It must be admitted that the paint film made from the same suspension is not uniform in thickness, and that the composition of the suspension is not, therefore, a true measure of the thickness of the film. This fact constitutes one source of inaccuracy in the results, but fairly consistent results have been obtained, especially when the readings from several samples have been averaged.

Two differences will be noted between curves for green, blue, and violet pigments on the one hand, and red, yellow, and orange on the other. The first applies especially to the dry pigments, The blue, green, and violet pigments are much the darker, not only because the lumino-

sity toward the violet end, where their maxima occur, is lower, but also because the maximum reflection is low. Lighter blues and greens exist, but their minima are correspondingly high. In other words, their increased brightness is at the expense of purity.

The second difference, applying both to dry pigments and to washes, is that greens and blues show a single peak, usually narrow, while reds, oranges, and yellows display a nearly horizontal line toward the red end of the spectrum, most noticeable in the washes, for the reason given. In other words, reds, oranges, and yellows have only one absorption band in the visible region, while greens and blues have two overlapping absorption bands. Of the two colors that belong to neither class, violets and purples have one wide absorption band leaving room for neither a peak nor a horizontal portion at the two ends, while yellow-greens show a gradual elimination of the absorption band on the right as the hue changes from green to yellow, the position of the peak remaining nearly constant.

There are no pigments (to the author's knowledge) that could be represented by a reversal of the forms of curves presented. If pigments are to be arranged in a continuous series instead of in two classes, it must begin with yellow, with the absorption band in the violet region, and as the band is moved toward the right the hue changes through orange to red and violet. The second depression appears as the hue passes through violet and blue and moves on to the right until green is reached. The transition from green to yellow would complete the cycle, but in fact the change is not a continuation of the movement of the peak and troughs, but a gradual elimination of the trough until the top of the yellow curve is reached. Curve *YG* in Fig. 2 is an illustration of the yellow-green intermediate type.

EXPERIMENTAL RESULTS WITH MIXTURES

The experimentally obtained curves for mixtures will be presented before considering theories and methods of calculation. Curves of the components will be given for comparison. A question arises here. Any mixture can be considered as an addition or as an average of its components; e.g., a wash containing two percent of a red and two percent of a yellow pigment can be considered as two percent of one added to two percent mixture of the other, or on the other hand as a mixture of equal parts of four percent mixtures of the constituents. If the reflections obeyed simple, definite laws, it would make no particular difference which method of comparison were used; but the average

method was chosen because it corresponds more nearly to practise, and the approximate relations can be more definitely settled. In some of the theoretical discussion, the mixtures will be considered as additions. Curves for the dilution of single pigments will also be included.

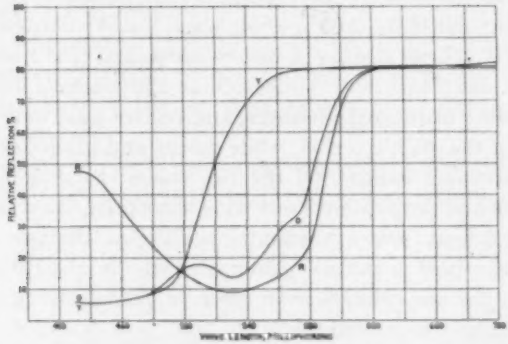

FIG. 5. *Mixture of red and yellow washes: R, geranium lake, 4%; Y, naphthol yellow, 4%; O, equal parts of R and Y.*

The mixtures were made, in some cases by mixing the suspensions of the two pigments, but more satisfactorily by making up new suspensions by weight. The pigments were mixed in simple ratios by weight

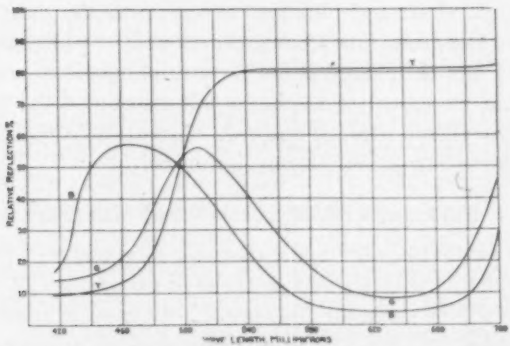

FIG 6. *Mixture of blue and yellow washes: B, peacock blue, 7%, Y, naphthol yellow, 1%; G, equal parts of B and Y.*

and in the curves shown for comparison, the mixtures correspond to equal parts of the suspensions, whatever their composition. The source of error already mentioned, the lack of uniformity of the film, applies both to the components and to the mixture.

In Fig. 5 are presented the curves for a geranium lake, and naphthol yellow *S* (a dye), both in 4 percent washes, and a mixture of equal parts of the two, i.e., a suspension containing two percent of each. Fig. 6 represents the same yellow in a 1 percent wash, a peacock blue lake in

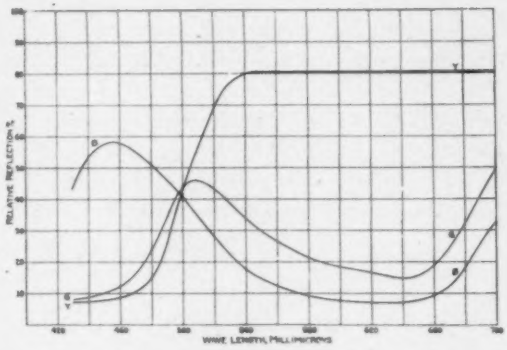

FIG. 7. *Mixture of blue and yellow washes: B, intermediate blue, 6%; Y, naphthol yellow, 2%; G, equal parts of B and Y.*

a 7 percent wash, and a mixture in equal parts making the final composition 0.5 percent yellow and 3.5 percent blue. In Fig. 7, the peacock blue is replaced by one intermediate between the peacock blue and blue

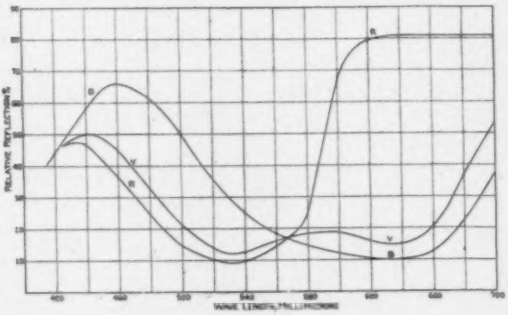

FIG. 8. *Mixture of red and blue washes: R, geranium lake, 4%; B, intermediate blue, 4%; V, equal parts of R and B.*

lake types represented in Figs. 2 and 4, and the constituent washes are 2 percent yellow and 6 percent blue. In Fig. 8 is a case of red and blue mixture, the red of Fig. 5 and the blue of Fig. 7 being used in equal parts.

Without any theoretical discussion, certain relations of the curves

are evident at once. Whenever the reflections of the two components are equal at the same wave length, for example at an intersection, the reflection of the mixture is approximately equal to either. It is evident that if the thicknesses of the paint films are equal, and their reflections are the same, that of the mixture must necessarily be the same. Such deviations as are found in actual measurements probably arise from non-uniformity of the film. Another noticeable point is that whenever the reflection of one component is widely different from that of the other, the reflection of the mixture is slightly above that of the component with the lower reflection.

The next field to be considered is that of mixtures of dry pigments. All measurements on such pigments are made on a layer thick enough

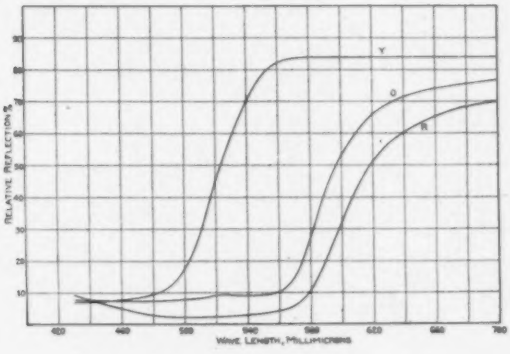

FIG. 9. *Mixture of red and yellow pigments: R, geranium lake; Y, pale chrome yellow; O, 2 parts of Y to 1 part of R.*

to be opaque, therefore corresponding to infinite thickness, and there is no question of thickness of layer. Moreover, one pigment replaces the other, rather than adding to the other, when a mixture is made, and the "average" method is the only one that can apply.

In Fig. 9 are shown the curves for a dry "geranium lake" (slightly yellower than the one used in the washes), "pale chrome yellow," and a mixture of one part of red to two parts of yellow. In Fig. 10 the curves are those of the same chrome yellow and a peacock blue.

The case of oil paints is very similar to that of dry pigments, the intervening spaces being filled with oil instead of air. Fig. 11 represents one of the few oil paint mixtures investigated, namely, the artist's colors, "cobalt blue" and "extra pale cadmium yellow," and a mixture

of equal parts of the two by weight. The mixture curve illustrates the results already shown. On the other hand, a mixture of a transparent red and a very dark blue give results not in agreement with the other or with expectations.

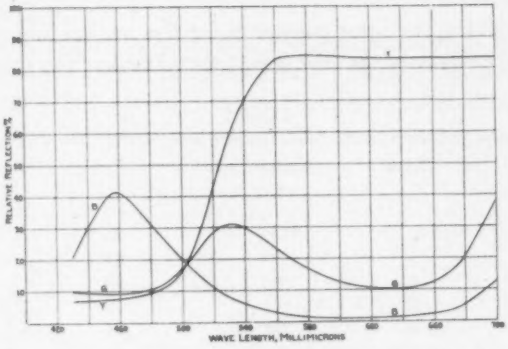

FIG. 10. *Mixture of blue and yellow pigments: B, peacock blue; Y, pale chrome yellow, G, equal parts of B and Y.*

In all these mixtures, then (except in such instances as the one just mentioned), the reflection of the mixture corresponds to that of either component at the intersection of their curves, and is slightly above that

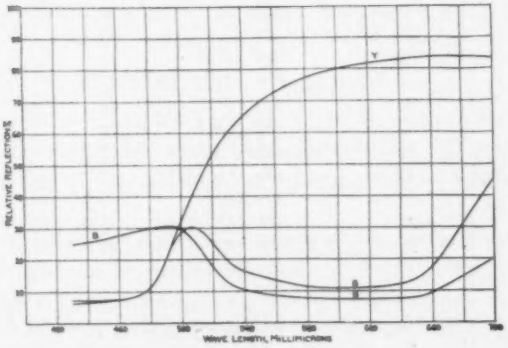

FIG. 11. *Mixture of blue and yellow oil paints: B, cobalt blue, Y, extra pale cadmium yellow; G, equal parts of B and Y.*

of the low component when the difference between their reflections is high. The result when one or both components are of the green-blue class is a mixture curve having a peak near the intersection and high points wherever both are fairly high. When both components are of

the red-yellow type the mixture curve has a steep portion intermediate between that of the two components. In the first case the mixture is darker than either component but purer than the single peaked component, in the second, the brightness and purity are intermediate between the two.

Table 1, giving the color constants for the washes and pigments in question, will illustrate the points just mentioned. (The terms are used

TABLE 1

Chart figure	Curve	Dominant wave length	Purity %	Brightness %
5,8	R	502.2 C	46 (C)	30.6
5	Y	579	88	67.2
5	O	592	78	36.4
6	Y	575	78	76.5
6	B	484	44	18.7
6	G	507.5	44	29.8
7	Y	576	83	73.3
7	B	479	32	17.2
7	G	545	52	28.0
8	B	477	30	23.2
8	V	400–430	5 ca.	17.1
9 (12)	R (A)	637	52.5	13.5
9, 10	Y	580	86	68.6
9	O	604	67.5	23.7
10	B	475	47	6.3
10	G	549	48	20.6
11	Y	579	84.5	66.5
11	B	479	23	11.9
11	G	550	46	15.6
12	B	640	25.5	8.8
12	C	635	43	22.8
12	D	493.5 C	40 (C)	30.8
12	E	495.2 C	32 (C)	40.5
12	F	499.9 C	24 (C)	52.0

in accordance with the recommendation in the Report on Colorimetry. The letter (C) following the percentage purity indicates that the method recommended for non-spectral colors has been used.)

In the hope of throwing more light on the effects of mixing, washes of different concentrations of the same pigment were applied and examined. Fig. 12 represents one of these, the geranium lake shown in Fig. 9. The solid pigment, the dry paint in the pan, and various washes are represented. The dry paint corresponds to a wash of infinite thick-

ness and therefore establishes a low limit for the reflection of the washes, practically reached in the 10 percent wash. At the other end, the reflection of the paper establishes a high limit (this pigment having lower reflection than the paper at all wave lengths) which is reached on the 1 percent wash (Curve *F*).

In this set of curves, a shift from spectral red into the non-spectral violet region in dilution is plainly marked. The increase at 430mμ is no greater than at 575mμ, at which points the reflections of the dry pigment are the same, but the violet component has more effect on the hue than the yellow-green.

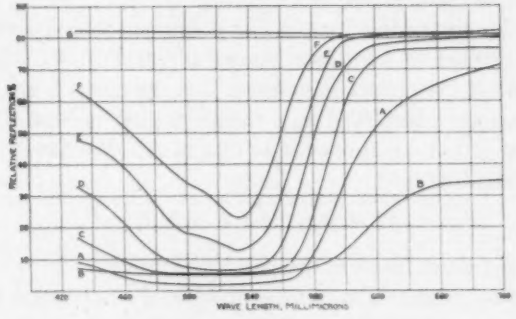

FIG. 12. *Dilution of geranium lake: A, dry pigment; B, dry paint in pan; C, 10% wash; D, 4% wash; E, 20% wash; F, 1% wash; G, paper.*

QUANTITATIVE RELATIONS

It has been shown that the general course of a curve for a mixture can be predicted from the curves of its components. It remains to be determined whether reasonably accurate quantitative relations can be found.

The laws of transmission of light are well known and simple. The fundamental rule can be expressed thus: if T_1, T_2, T_3, etc., are the transmittances[11] of any transmitting layers, for homogenous light of any given wave length, the transmittance, T_m of the layers superimposed, is

[11] The terms and symbols used, as far as possible, are in accordance with the recommendations of the Committee of Spectrophotometry, J. O. S. A. & R. S. I. *10*, 177. In the absence of a full system of conventions for the more complex case of reflecting surfaces, the term reflectance, with the symbol ρ, has been used to denote surface reflection, and the term reflection, denoted by *R* and *r*, for the reflection as a whole, including reflection and transmission factors; in other words, the ratio of the radiant energy going out from the upper surface to the radiant energy incident on the same surface. The symbol *d* has been used for thickness of the paint film and *a* and *b* reserved to distinguish the paints.

$$\top_m = \top_1 \top_2 \top_3 \cdots \tag{1}$$

In the special case of a homogeneous substance, we may adopt the symbol t for transmissivity or transmittance of unit thickness, and d for the thickness of the layer, and we have Lambert's law in the form

$$\top = t^d. \tag{2}$$

As a first approximation to the conditions of painted surfaces, it may be assumed that the surface reflection is negligible and that the film of a single paint is homogeneous. Moreover, it may be assumed throughout the discussion that the unpainted surface (the paper) is smooth and has a reflection R_p.

Let t_a be the transmissivity of a film of homogeneous paint A (of unit thickness) applied to a surface having a reflection R_p, and let r_a be the reflection of the surface so painted. Let the terms t_b and r_b apply to a film of another paint B of unit thickness (not necessarily the same thickness by actual measurement as that of A). We have the relations for normal incident light,

$$r_a = R_p t_a^2 \text{ and } r_b = R_p t_b^2, \tag{3}$$

since the light must traverse the paint film twice.

Now suppose a paint film to be composed of a units of A and b units of B. Since we are neglecting reflectance at all surfaces, it matters not whether thick layers are superimposed or the whole is thoroughly mixed. If R_m is the reflection of such a mixture, we have

$$R_m = R_p t_a^{2a} t_b^{2b}, \tag{4}$$

which reduces on substitution to

$$R_m = R_p \frac{r_a^a}{R_p^a} \frac{r_b^b}{R_p^b} = R_p^{1-(a+b)} r_a^a r_b^b. \tag{5}$$

Two special cases may be noted. First, in case of dilution of a single paint, $b=0$, $r_b^b=1$, and

$$R_m = R_p^{1-a} r_a^a. \tag{6}$$

Second, in case the paints are mixed before applying, so that the film remains of unit thickness, $a+b=1$, and therefore

$$R_m = R_p \top_a^{2a} \top_b^{2(1-a)}, \tag{7}$$

which reduces to

$$R_m = r_a^a r_b^{1-a}. \tag{8}$$

A nearer approach to actual conditions may be made by taking reflectance at the surface of the film into account, while retaining the assumption that reflection at bounding surfaces within the paint is negligible.

Fig. 13 represents a cross-section of a uniform paint film on a smooth surface, and AB an incident ray of monochromatic light. (The rays are assumed to be normal although necessarily represented as oblique.) At B a fraction, ρ, of the light is reflected, independently of wave length. The fraction $1-\rho$ traverses the film and the portion reaching the lower surface is $(1-\rho)\top$. Of this light a fraction R_p is reflected at the lower surface at C, and of the reflected light a portion $\top$ is transmitted to the upper surface. Therefore the fraction of the original light reaching the

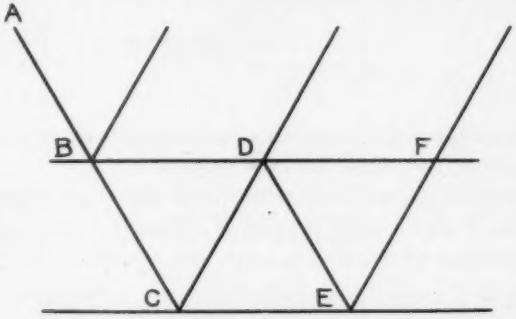

FIG. 13. *Theoretical cross section of uniform film of paint. BDF, upper surface; CE, lower surface; AB, incident light.*

upper surface at D is $(1-\rho)R_p\top^2$. Of this light, a fraction ρ is reflected back and the remainder $1-\rho$ passes out. The second component of the external reflection, then, is $(1-\rho)^2R_p\top^2$. The reflected portion $\rho(1-\rho)R_p\top^2$, on its second course DEF, is subject to the same deductions as before, and the portion passing out at F is $\rho(1-\rho)^2R_p^2\top^4$. At each successive cycle of reflections, the additional factor $\rho R_p\top^2$ is introduced, and the sum of the external reflections (n being the number of the term) is

$$R = \rho + (1-\rho)^2R_p\top^2 + \rho(1-\rho)^2R_p^2\top^4 + \rho^2(1-\rho)^2R_p^3\top^6 \cdots + \rho^{n-2}(1-\rho)^2R_p^{n-1}\top^{2n-2}. \quad (9)$$

The surface reflection, ρ, is of the order of magnitude of 0.04. Calculations on actual cases agree with the result of inspection of the formula, that the third term is not negligible when $\top$ is large, but that the maximum possible ratio of the fourth term to the second is 0.001.

Therefore, all terms beyond the third can be neglected, and the equation becomes.

$$R = \rho + (1 - \rho)^2 R_p{}^2 \top + \rho(1 - \rho)^2 R_p{}^2 \top^4. \tag{10}$$

If the film is of unit thickness, and the same terms as before are used,

$$R = \rho + (1 - \rho)^2 R_p t^2 + \rho(1 - \rho)^2 R_p{}^2 t^4. \tag{11}$$

Solving for t^2, we have

$$t^2 = \frac{\sqrt{1 + \dfrac{4\rho(r - \rho)}{(1 - \rho)^2}} - 1}{2\rho R_p}. \tag{12}$$

It will simplify the following expressions if we use the term

$$m = 2\rho R_p t^2 = \sqrt{1 + \frac{4\rho(r - \rho)}{(1 - \rho)^2}} - 1, \tag{13}$$

with the subscripts a and b denoting the components. The quantity m must of course be calculated for each value of r.

Again the most general case is that of a paint film composed of a units of paint A and b units of paint B. Then $\top_m$, the transmittance of the combination or mixture, is $t_a{}^a t_b{}^b$, and equation (10) becomes

$$R_m = \rho + (1 - \rho)^2 R_p t_a{}^{2a} t_b{}^{2b} + \rho(1 - \rho)^2 R_p{}^2 t_a{}^{4a} t_b{}^{4b}, \tag{14}$$

or, more conveniently,

$$R_m = \rho + (1 - \rho)^2 R_p t_a{}^{2a} t_b{}^{2b}(1 + \rho R_p t_a{}^{2a} t_b{}^{2b}), \tag{15}$$

since the quantity in parenthesis is a factor often differing little from unity, and the difference is negligible in some cases. By substitution we have

$$R_m = \rho + \frac{(1 - \rho)^2 R_p m_a{}^a m_b{}^b}{(2\rho R_p)^{a+b}} \left(1 + \frac{\rho R_p m_a{}^a m_b{}^b}{(2\rho R_p)^{a+b}}\right). \tag{16}$$

For the dilution of a single pigment, $b = 0$, and the formula reduces to

$$R_m = \rho + (1 - \rho)^2 R_p \left(\frac{m_a}{2\rho R_p}\right)^a \left[1 + \rho R_p \left(\frac{m_a}{2\rho R_p}\right)^a\right], \tag{17}$$

and for the case of a mixture of unit paints, $a + b = 1$, and therefore

$$R_m = \rho + \frac{(1 - \rho)^2}{2\rho} m_a{}^a m_p{}^{1-a} \left(1 + \frac{m_a{}^a m_b{}^{1-a}}{2}\right). \tag{18}$$

As an example of application of this theory to dilution, in Fig. 14 the full lines are the spectrophotometric curves for a series of washes of geranium red (the same paints as plotted in Fig. 12, on a logarithmic scale, but including later tests). From each series, log t^2 was calculated for a number of significant points, and from the mean value the reflections were again calculated, as represented in the dotted lines.

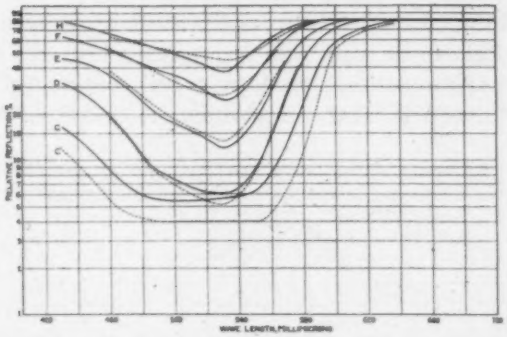

FIG. 14. *Dilution of geranium lake washes on logarithmic scale. Full lines: C, 10% wash; D, 4% wash; E, 2% wash, F, 1% wash (as, in Figure 12), H, 0.5% wash. Dotted lines C and unlabeled, are calculated values for the corresponding washes.*

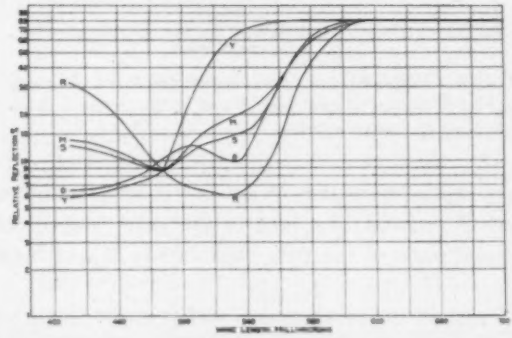

FIG. 15. *Mixture of red and yellow washes, logarithmic scale; R, geranium red, 4%; Y, naphthol yellow, 4%; O, mixture of equal parts of R and Y, equivalent to 2% of each component; M, geometric mean of R and Y; S, calculated from R and Y with allowance for surface reflection.*

In the absence of reliable information on the refractive index of dextrin or the mixture used in the paints, its surface reflectance was taken as 0.04, corresponding to that of glass. Probably the actual value is lower. An assumed value of 0.05 would have given closer correspon-

dence to actual results, but washes of other colors have a lower minimum reflection than 0.05.

The inaccuracy due to an insufficiently large number of observations and lack of control of thickness of the paint film must be noted again, both in the experiments on dilution and on mixtures. It is evident, however, that correction for internal reflection would bring the curves into better conformity with experimental values.

In Fig. 15 the same component washes and mixture as in Fig. 5, are shown on a logarithmic scale, with two calculated curves. Curve *M* is the geometrical mean of the component curves, uncorrected for surface reflection. Curve *S* is calculated according to equation (18). In Fig.

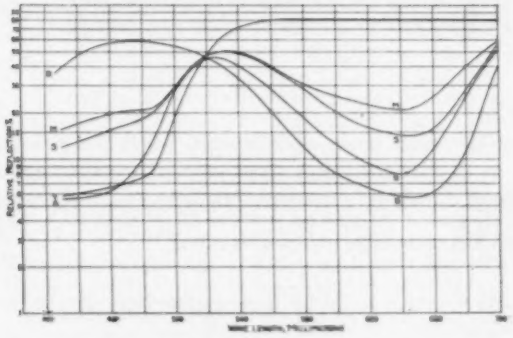

FIG. 16. *Mixture of blue and yellow washes, logarithmic scale: B, peacock blue, 4%; Y, naphthol yellow, 4%; G, mixture of equal parts B and Y; M, geometric mean of B and Y; S, calculated with allowance for surface reflection.*

16, the curves represent washes of a 4 percent suspension of naphthol yellow *S*, one of peacock blue lake of the same concentration, and a mixture of equal parts of the two, with curves *M* and *S* calculated as before.

In Figs. 17 and 18 the two percent washes of the single pigments are regarded as the components of the mixture, and curve *C* is that calculated by formula 16.

Before discussion of these results in full, Appel's[12] method remains to be considered.

In this method, no mathematical basis of calculation is used, but the procedure includes the following steps:

[12] Loc. Cit.

(1) A concentration-reflection curve is plotted for each wave length to be investigated.

(2) The reflection of component *A* is read from the curve, and the same reflection is located on the corresponding curve for component *B*.

(3) To the concentration of *B* giving the reflection in question, the

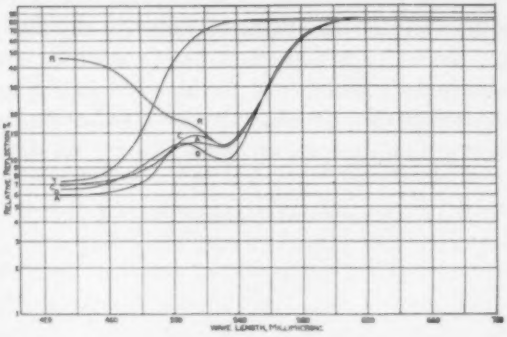

FIG. 17. *Addition of red and yellow washes. R, geranium lake, 2%; Y, naphthol yellow, 2%; O, geranium lake and naphthol yellow, 2% each (same as O in Fig. 15); C, calculated by formula 16; A, calculated by Appel's method.*

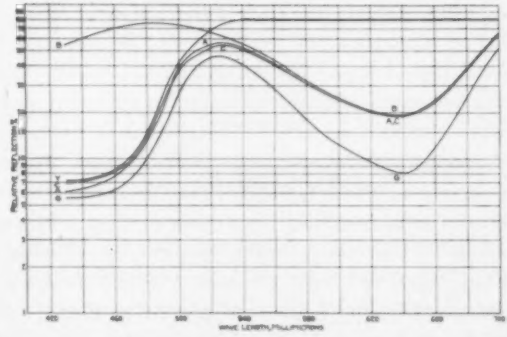

FIG. 18. *Addition of blue and yellow washes: B, peacock blue, 2%; Y, naphthol yellow, 2%; G, peacock blue and naphthol yellow, 2% each (same as G, in Figure 16), C, calculated by formula 16; A, calculated by Appel's method.*

actual concentration of *B* is added, and the reflection corresponding to the total concentration is taken to be that of the mixture.

Over the range of Appel's published experiments, the concentration reflection curves for dyes on a logarithmic scale are linear, and the calculated result corresponds closely to observation, whichever component

is chosen as reference. In the case of the concentrations used in these experiments, however, the relation is less regular, as will be seen on inspection of Fig. 19, in which the data of Fig. 14 are plotted on a concentration reflection scale.[13] Sometimes the process can be applied in only one direction. When both are possible, the results do not always agree. Curves *A* in Figs. 17 and 18 are derived by Appel's method representing the mean of the results obtained by the two procedures.

In Fig. 18, either Appel's method or the addition method gives reflections very little below that of the blue component in the region where the yellow component is almost transparent, and it is natural to expect that the yellow component would deduct but little from the

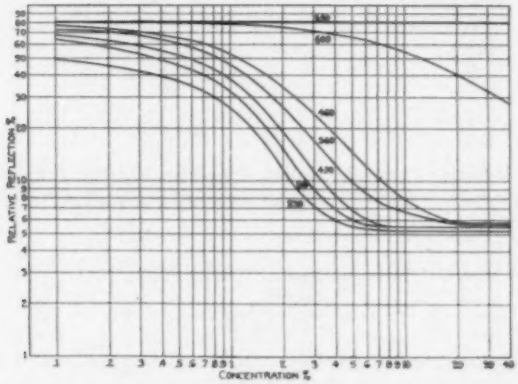

FIG. 19. *Concentration—reflection curve for geranium red washes. Figures indicate wave length for corresponding curve.*

reflection. The question is thus raised, whether the average result used for the green mixture curve is too dark to correspond to a true mixture of the components as represented. The answer is, that if a lighter mixture were chosen, such that its reflection through the yellow, orange, and red regions were close to that of the blue component, the reflection at 510 to 540mμ would be higher than that calculated by any method. On the other hand, if the blue wash were somewhat stronger, the calculated curves would lie nearer the actual green curve between 520 and 700mμ. The blue curve as obtained is reasonably consistent with that of other washes of the same pigment, but the yellow pigment used, being a dye,

[13] Curves of the same general form have been determined by Appel at correspondingly high concentrations, and are in accordance with theory.

has been shown to be more effective in proportion to its weight than the blue pigment.

By comparison of the four sets of curves, namely, those in Figs. 15 and 17 for red and yellow, and in Figs. 16 and 18 for yellow and blue, it will be noted that a closer approximation to the actual curve is obtained by considering the components as added than as averaged. Moreover, the results obtained by Appel's method have been found to agree closely with those calculated from the "added" components. Appel's method has the theoretical advantage of obviating the necessity of introducing the internal reflection into the calculations.

The most important factor neglected in all the above calculations is surface reflection within the pigment layer. Such reflection takes place at the surface of each particle of pigment, whether in contact with binder or air, and binder-air surfaces may also be present. As an approximation to the conditions just outlined, the author has considered the case of alternate layers of pigment and binder, all parallel to the upper and lower surface of the entire film. It appears possible to develop a formula giving a sufficient approximation without using terms of higher degree than second, but in order to make practical computations it would be necessary to know accurately the refractive indices of the pigment and binder and particle size. Naturally the thicker the film and the finer the pigments, the greater part the internal reflections play.

The limiting case is reached when the layer is sufficiently thick to cause extinction of all wave lengths before the lower surface is reached. That is the situation in dry pigments in the condition in which they were studied, and also in oil paints in sufficiently heavy coats. In the first case, the bounding surfaces are all air-pigment; in the second, oil-pigment if the paint dries without cracking or formation of bubbles. Since a large number of particles and interspaces must be traversed by the light before extinction occurs, internal reflection must play a large part.

For the reason just given, calculation of reflections of mixtures from those of their components becomes more difficult. Qualitatively, the same principles should apply as in the case of washes; for instance, if the reflections of the two components at a given wave length are equal, the reflection of the mixture should be equal to either, unless the two vary widely in particle size, covering power or some other property.

The data given confirm this expectation. Quantitative prediction, however, is difficult, because we have as data only the reflections corresponding to infinitely thick layers of non-uniform air or oil with the separate components and that of a similar mixture of the two for com-

parison. As before, in calculations made on the assumption of a mixture without internal reflection, weight may be used as well as any other property as the basis of composition.

Known mixtures of iron oxide with white pigments were examined, and also a few mixtures of geranium red and of English vermilion with a white pigment. In the case of iron oxide, a fair approximation to the curve for a given mixture could be made by calculating the quantity a in equation 6 from the data for pure iron oxide and for the mixture, then calculating the reflections for the different wave lengths by the use of the mean value of a. However, the figures thus obtained were of no assistance in calculating the reflections of mixtures in other proportions. The actual reflections always fall below those calculated on the basis of proportions by weight or on the basis just described, with a mixture richer in the colored pigment. In the cases of more intensely colored pigments, the mean value of a for a given mixture did not even yield consistent results for the same mixture.

Bruce[14] has developed an empirical formula for the brightness and purity of white and colored pigments in oil. Applied to the brightness of the dry mixtures of iron oxide and white pigments, the formula gave results with about the same order of accuracy as in Bruce's own experiments, and the deviations were not much greater when the formula was applied to reflections at individual wave lengths, although it does not follow from the formula that it is so applicable. With the more intensely colored pigments, e.g., geranium lake, Bruce's formula fails, as far as individual reflections are concerned.

PRIMARY COLORS

Primary art colors, i.e., colors that may be used in any form of art involving pigments, may be defined in the broadest sense as the smallest group of colors that may be combined to produce all possible hues in a satisfactorily high degree of brightness and purity. Since in all common forms of art pigments in some form are employed single or mixed to produce the desired effect, the definition may be applied to pigments by restating it in the following form: Primary art colors are the colors of the smallest number of pigments that may be mixed (according to the methods of the particular art in question) to produce all possible hues in a satisfactorily high degree of brightness and purity. Black and white pigments, which may be added to decrease the purity and vary the brightness, are not ordinarily considered primaries.

[14] Loc. Cit.

The word "satisfactorily" is advisedly indefinite, to the extent that the approximation to the purest obtainable color depends upon the purpose of the art work. At one end, leaving out of consideration monochrome pictures, which are not intended to represent objects in natural colors, the lowest number of primaries must be two. The most common illustration of a two-color system is that used in colored motion picture films, in which the brightness of blues and yellows is sacrificed to the necessity of using two primaries. Still, the result is satisfactory to the extent that the great majority of motion picture patrons do not realize the deficiency in color until their attention is called to it. Color in motion pictures is secondary to action. In most forms of art, however, contemplation, or at least continued attention, is one purpose of the art object and blue and bright yellow would quickly be missed in a picture intended as a stable representation of reality. Therefore for most forms of color art, especially painting, the number of primaries must be at least three.

The choice of primaries could be made on the assumption that a pigment existed corresponding to any possible spectrophotometric curve, and the author has made some attempts to solve the problem from that standpoint, but it is evident that each kind of mixing, i.e., washes, oil paints, etc., has its own intricacies, and the data for water color washes and oil paints, at least, are not yet sufficiently accurate. The principle that "The subtractive primaries are the additive secondaries" implies that pigments corresponding to the additive secondaries are available, but such is not strictly the case. The subject will be considered at present, therefore, only from the standpoint of available pigments.

Actual pigment colors have been classified as "plus" and "minus" colors, according to the form of their curve, a "plus" color being one that owes its hue largely to its maximum reflection in the corresponding part of the spectrum, and low reflection in all others, while "minus" color is deficient in reflection of its complementary hue, but reflects other wave lengths well. The actual curves cannot be entirely classified under these headings. Yellow, as represented in most pigments, is outstandingly a "minus" color. Some greens represent the "plus" type, but practically all others are intermediate.

Yellow must be a primary color, because a yellow having any of the curves presented cannot possibly be duplicated by any subtractive mixture. The best yellow that could be produced by mixing even an orange and a yellow green would have a peak much below the maximum

reflectance of a bright yellow, and would therefore be so lacking in brightness that it would be in common language olive or tan.

Orange cannot be a primary, because it could not yield a good yellow, and from the position of its curve between the curves for yellow and red, can easily be made by a combination of yellow and red.

Green can yield only mixtures varying from green yellow to green blue, as its combining power is limited by the sharpness of the ascents of its peak.

On the other hand, although blues have nearly as sharp peaks as greens, a blue can be chosen, as shown in the charts, so that its intersection with yellow can still yield a green comparing well with the natural greens. On the other side, it can combine with blue violet and violet to form intermediate hues.

Violet occupies a peculiar position. The violet, purple, and magenta or violet-red pigments are characterized by high reflection at the ends of the spectrum, the hue depending on the relative reflection of the red and violet ends. The only combination of pigments that would duplicate any given curve would necessarily consist of two of the same general type, one higher in the red, and the other lower, and reversing the order at the violet end. The extreme possibilities, then, must be a blue with a high reflection in the red, and a red high in violet. If the number of primaries is to be limited to three, these extremes must be used, because the yellow-red combination will be spoiled by too high an intersection in the blue region, and the blue-yellow combination is even more delicate. As seen in Figure 8, the violet produced by the mixture of the pigments shown is satisfactory for many kinds of work, though not so bright as the best single violet pigments.

The limitation imposed on the red has already been indicated. It must have a high enough violet component to yield a fairly bright violet or purple mixture, and still not so high as to spoil the red-yellow combinations. This limitation makes the red necessarily a non-spectral color, and possibly violet-red would be a better designation for it. That nomenclature would have the advantage of deviating from the Brewsterian system of red, yellow, and blue. The dry pigment may be a spectral red, because the violet component increases markedly on dilution.

Returning to the blue, the additional limitation, that it must have a high red component, can easily be accepted, because the reflection in the extreme red of the blues studied has very little influence on the dominant wave length.

Comparing these primaries with the theoretical "additive secondaries" or rather the theoretical "minus" primaries according to the Young-Helmholtz theory, we find the yellow to correspond to the minus violet. A blue or "minus red" would be either one in which the reflection at the red end is low, or one having a curve with a minimum at the point of maximum red excitation and rising on both sides. Actual blues differ in having the long-wave end of a second absorption band in the violet and a wide low point in the orange, but the wave length at the minimum reflection corresponds nearly with that of the theoretical curve. The practical violet-red primary is more red than a "minus green" having a minimum at 500 to 520 $m\mu$, but the latter color would not mix very well with yellow.

The brightness and purity of the mixtures would naturally be improved by the addition of more primaries. If a fourth is to be added one of two procedures can be followed. A green can be added, the blue replaced by a more nearly violet one, and the violet red left substantially as it is, or medium purple (about midway between red and violet) can be added, the blue replaced by a more greenish blue, and the red shifted to more nearly a spectrum red. A still better combination would be to add both a purple and a green, leaving the other colors nearly as they are, thus closely approaching the Munsell system. The omission of orange in that system, although resting on other bases, is given the additional justification that mixtures of red and yellow nearly equal to the single oranges may easily be made.

For more accurate color work, the number of primaries must be increased even beyond five. Artists differ in their needs in that respect. In elementary color work, however, where economy is a factor, the use of these colors provides a very satisfactory system.

CONCLUSION

It has been demonstrated that the results of the mixing of paints (especially water colors) may be predicted roughly by determination of the spectrophotometric curves of the constituents, approximate derivation of the curves of the mixture through the application of certain general principles, and determination of the color of the mixture from its curve. An accurate prediction of the curve of the mixture cannot be made with data and methods now available.

It has been shown that the smallest number of practical primaries must be three, and that they must be blue, yellow, and violet red.

The experimental work has been supported and the publication of results has been made possible by the American Crayon Company.

The author is indebted to Dr. D. B. Judd for valuable assistance in connection with the mathematical part of the paper.

A SUGGESTED METHOD FOR APPLYING THE SLITLESS SPECTROGRAPH TO THE MEASUREMENT OF THE DOPPLER SHIFT

By Noel Deisch

[908 G St., Washington, D. C. Received Aug. 21, 1930]

In Fig. 1 is shown a stellar spectrograph of the objective prism type adapted to be worked in either of two operative settings, the change

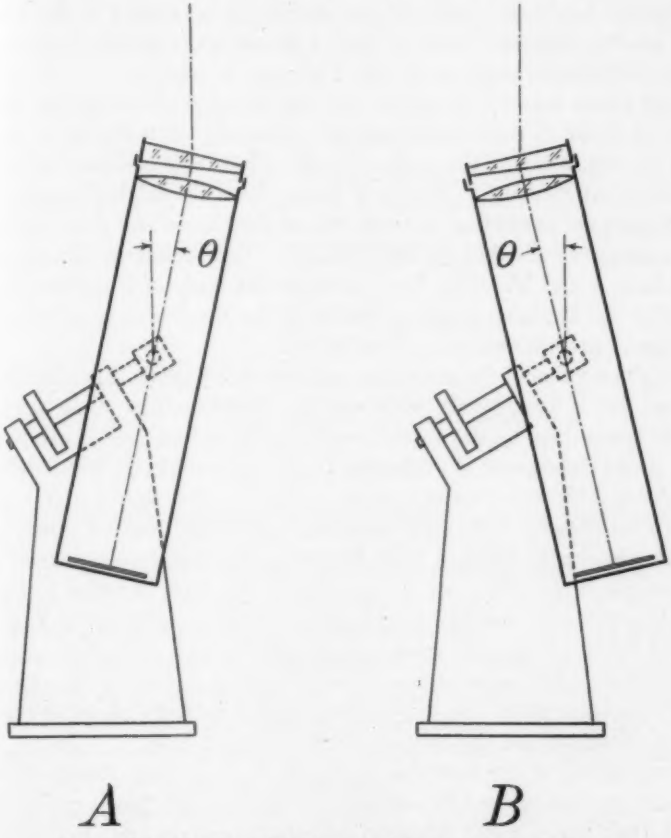

Fig. 1. *Objective prism stellar spectrograph in two operative settings.*

from the one to the other setting being made by reversing the prism and shifting the telescope on its declination axis through an angle of 2θ

where θ corresponds to the deviation at a mean wave length of the projected spectrum. The image formed in the focal plane of the instrument when in the position A contains a field of star spectra the members of which have the same positional relation as when the instrument is in the position B, but with the difference that the spectra corresponding to the two positions have their long-wave and short-wave terminators in inverse relation. Suppose a spectrogram be made by the usual "drift" method with the instrument in the position A, and that after the exposure has been completed the instrument be shifted to position B and another exposure made to form a second spectrogram. Contiguous spectra related as shown in Fig. 2 at A are obtained.

In this figure λ and λ' represent the respective positions in the two spectra of a line of given wave length. A Doppler shift displaces this line in the two spectra in opposite directions into new positions shown respectively at λ_1 and λ_1', distant d and d' from the original position. The distance D' separating the two shifted lines λ_1 and λ_1' thus differs by an amount $d+d'=2d$ from the distance D that separates the undisplaced lines λ and λ'. This fact furnishes the basis of a method for measuring the Doppler displacement in stellar spectrograms obtained by means of an objective prism instrument.

For a given value of the reversing angle 2θ the distance D of any undisplaced line is fixed by the scale and the dispersion law of the spectrum, as determined by the optical constants of the instrument. Hence for any given instrument the values of D for various critical lines might be established by direct measurement, using as a test source a star or stars of zero Doppler shift. The resulting data would make it possible to obtain the double Doppler shift $2d$ from simple measurement of D', a known line in the spectrum of some one of the stars included in the field, when this spectrum had been independently measured as to Doppler shift by means of a slit spectrograph, serving as a control to show the correction, if such were necessary, to be applied to each of the other stars on the same plate due to systematic errors introduced by the reversal of the prism and the shifting of the telescope tube.

The magnitude of the reversing angle, which determines the relative position of overlap of the two fields, is not rigidly fixed, but is optional within certain limits, and it might be desirable in certain cases to choose this angle as one which would bring selected stellar lines in the two spectra into close parallel relation, as shown at B, in Fig. 2, where the distance D for the lines λ and λ' has been reduced to zero. By this means measuring errors, and plate errors due to displacement of the

emulsion during processing, would be diminished. It might further be of advantage to suppress, by means of a monochromatic filter, all but the critical portion of one of the spectra, as shown in Fig. 2 at *C*, to minimize accidental overlapping with spectra of neighboring stars.

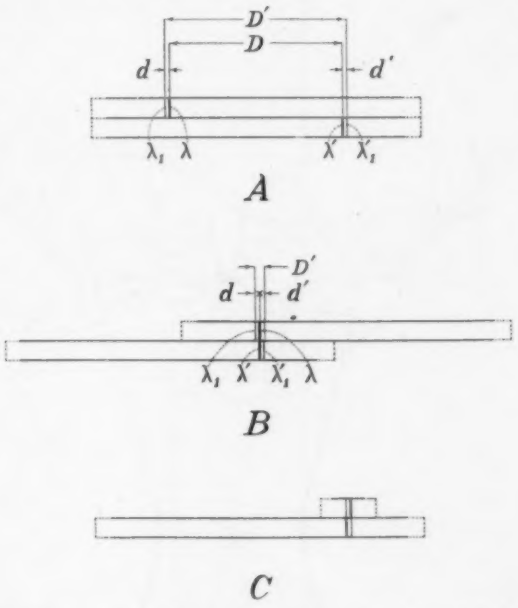

FIG. 2. *Spectrograms obtained by successive exposures in positions* A *and* B *of Fig. 1.*

The mechanical gear involved in carrying out the method is quite simple. The prism or prisms might for the purposes of reversal be mounted in a cell rotatable about the optical axis of the telescope, as shown in Fig. 1, stops being provided to facilitate locking the cell in positions of the proper azimuth. The value of the reversing angle would likewise be fixed by abutment stops and clamps, these held by a support moved by the declination gear coaxially with the declination shaft, the declination shaft itself being free.

A composite instrument comprising two identical optical systems mounted respectively in the positions *A* and *B* of Fig. 1 might of course be used, the displacements recorded on the two plates being measured by means of a comparison microscope. This arrangement is shown in Fig. 3. Changing the setting during exposure would be avoided, and

the length of the exposure for a given aperture reduced by one-half. Moreover, overlapping of adjacent spectra would be minimized, this being of advantage where dense fields of stars were being photographed.

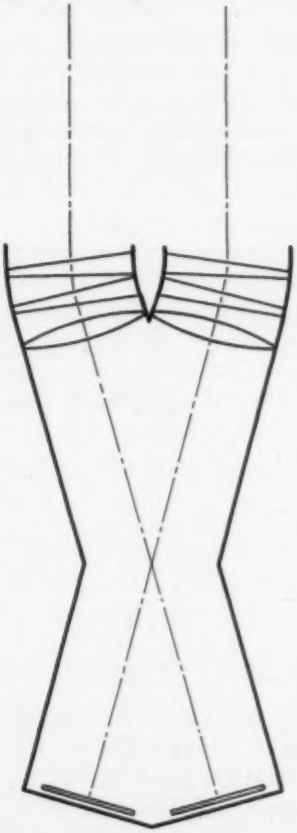

FIG. 3. *Composite instrument containing two optical trains in positions* A *and* B *of Fig. 1.*

An alternative arrangement of the instrument in which but one plate is used and yet the necessity for changing the mechanical setting between exposures avoided, is shown in Fig. 4. The instrument comprises two prism trains arranged in inverse relation, each train serving to introduce a beam of rays into separate halves of the objective. The object glass is cut in two on a diameter falling in an extension of the plane of adjacency of the two prism trains, and the two halves of the

divided lens are so mounted that their respective optical axes diverge by a very small angle, by which means the spectra formed by the two prism trains are given a slight separation in the focal plane. By allowing

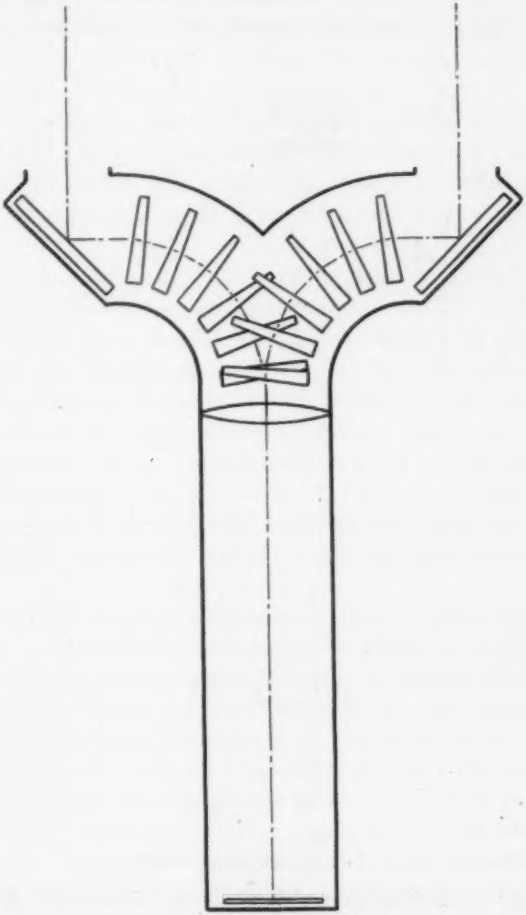

FIG. 4. *Spectrograph containing two prism trains forming double reversed spectra.*

the telescope to drift during exposure an amount equal to this spacing interval, spectra of the type shown in Fig. 2 result. With this arrangement systematic changes in the value of D would be very greatly reduced, but the instrument would be optically inefficient and costly.

The feasibility of using as a control spectrum for the determination of D a spectrum produced from a local source and impressed at some convenient position on the recording photographic plate along with the steller spectra, is contingent in the above instruments on the practicability of producing a collimator which will maintain two beams of

FIG. 5. *Collimator for producing spaced parallel beams.*

light in mutually parallel relation without loss of adjustment over considerable periods of time. The accessory optical system may be arranged as a static unit mounted separately from the spectrograph and adapted to impress the control spectrum by a short exposure immediately preceding or succeeding the exposure to the star field proper, or it may be attached to and share the motion of the spectrograph, the two exposures in this case preferably taking place simultaneously.

In the double collimator diagramed in Fig. 5 the prisms P and P' must be supposed to overlie the two objective prism apertures of the spectrograph. The slit S would under these conditions be imaged as a spectrum on the recording photographic plate by each of the branches of the spectrograph, and by properly orienting the axis of the illuminator with respect to the axis of the spectrograph these spectra could be made coordinately to take up any desired position on the photographic plate. In applying the collimator to instruments of the type shown in Fig. 1 and Fig. 4 it would of course be necessary to cause the beams A and A' to diverge at a small angle in a plane perpendicular to the plane of the drawing, so as to secure a proper separation of the two spectra.

The slit S is adjustable along the optical axis of the collimator to secure parallelism of the emergent beams A and A'. Strictly speaking it is of course not essential that the two beams be truly parallel, a deviation from parallelism simply introducing a correction factor. The indispensable condition is that a relative angle, if such exist, be permanent, the limits as to stability of adjustment being very close indeed. Continuance of adjustment in the above instrument rests in greater part on maintaining the given relation between the objective and the slit, since due to the constant deviation feature of the penta prisms slight changes in their adjustment would not appreciably affect parallelism.

The method of determining D from a control spectrum produced locally as outlined above would seem to be subject to the limitation that, though wandering of D caused by displacements in the optical system of the stellar spectrograph proper would be revealed, other errors might be introduced by the collimator itself, due to systematic changes in its optical structure following from deformation of the mounting. The local control spectrum could hence be used only in the way of a secondary standard, to be checked at intervals against the known spectrum of a suitable star. This condition applies to any type of spectrograph in which the objective prism aperture is displaced between exposures, as with the instrument shown in Fig. 1, or in which two objective prism apertures exist, as with the instruments shown in Figs. 3 and 4.

It is however possible to modify the design of the type of spectrograph shown in Fig. 1 in such a way that the objective prism aperture would not be displaced during the change of setting, and in this case a single-beam collimator could be used. If this last were mounted separately

from the spectrograph, to avoid possibility of its being displaced during the shift of the spectrograph setting, the control exposures being made in close succession immediately before and after the shift, collimator errors would be reduced to a very low order.

THE ABSORPTION OF RADIATION FROM DIFFERENT SOURCES BY WATER AND BY BODY TISSUE

By W. E. Forsythe and Frances L. Christison

[Incandescent Lamp Department, General Electric Co., Nela Park, Cleveland, Ohio. Received Sept. 6, 1930]

There are several infrared or total radiation devices on the market intended for therapeutic purposes which use as the source of the radiation, heaters that vary from low temperature blackened iron surfaces to high temperature tungsten lamps. Claims are often made for the value of this radiation without considering certain physical limitations. When radiant energy from a source of continuous radiation falls upon any substance in general, part is reflected, part transmitted and a part is absorbed. The percentage of the incident light reflected, transmitted or absorbed depends upon both the characteristics of the substance and the wave-length distribution of the incident radiation. Thus, in the use of these infrared radiators it is important to know whether the radiation is to be absorbed in a very thin layer on the surface of the body or whether it is desired to have the radiant energy penetrate deep into the tissue, because as shown in what follows, it is possible by the selection of the source of radiation to determine whether the radiation that is not reflected from the surface shall be absorbed in a very thin layer near the surface or whether a considerable portion of it shall penetrate deep into the tissue of the body.

The data that are given in this paper show that almost all of the radiation that penetrates the surface from low temperature sources will be absorbed in a very thin outer layer of the body, while a considerable portion of the radiation from the high temperature sources that is not reflected from the surface may penetrate deep into the body tissue. This is due to the fact that a high temperature source radiates a larger portion of its energy in the wave-length region where the transmission of the body tissue is at its maximum. The body is known to be made up of a large percentage of water and thus it can be said that radiation that is not transmitted by water will not penetrate very far into the body.

The spectral transmission curve for water is known and in Fig. 1, curve *B* is shown the transmission for different wave lengths for one centimeter of water and in curve *A*, the spectral transmission for one millimeter of water. The marked difference between these curves which is particularly noticeable for wave length about 16000A, is due entirely

to the different thicknesses of water considered in the two cases. Water has other transmission bands for longer wave lengths than those shown in Fig. 1, however, the value of the transmission for the long wave-length tramsmission bands is only a small fraction of one percent. Thus the curves shown in Fig. 1 can be said to represent the wave-length regions for which a path of water one centimeter long or one millimeter long will transmit any appreciable part of the incident energy.

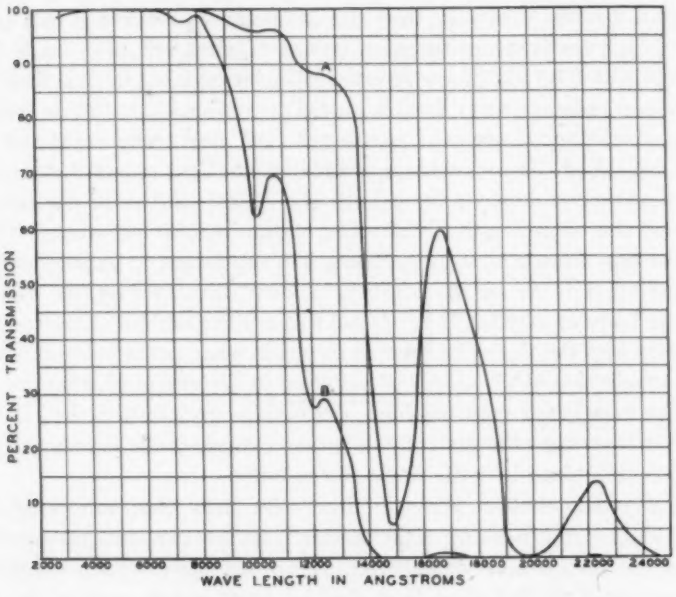

FIG. 1. *Spectral transmission of water.*
Curve *A*—*Transmission for a thickness of one millimeter.*
Curve *B*—*Transmission for a thickness of one centimeter.*

Carthwright[1] has given some data on the transmission of the human cheek for radiation of different wave lengths. His curve is reproduced in Fig. 2, curve *B*. The second curve *A* given in Fig. 2 represents the transmission factor of the human cheek. These values were obtained from curve *B* by applying a correction at each wave length of 34% which is the value given by Cartwright as the reflection factor for this wave-length region for the surface of the body. In using these values the

[1] J.O.S.A., *20*, 81; 1930.

assumption has been made that the transmission of the cheek was uniform through both the skin and the tissue. This assumption and the correction for reflection may not be absolutely correct but this error will probably not change the final conclusion any very great amount.

From what has just been said it can be seen from the curves in Figs. 1 and 2 that energy of wave lengths longer than 14000A cannot penetrate very far into the body. Energy of wave lengths longer than this will be partly reflected from the surface and partly absorbed in a very thin layer with the result that the surface temperature of the body will be raised. This accounts for the fact that radiant energy from a low temperature source seems much hotter than the same amount of energy from a high temperature source. An interesting illustration of this is the following: If a nickel plated electric flat iron operating at normal temperature is held within three or four inches of the face the radiation from it seems very hot, much more so than the radiation from the sun in midsummer. In midsummer with the sun directly overhead one square centimeter of exposed surface receives radiant energy at the rate of about 107 milliwatts, while with the heated electric iron held within three or four inches of the face, the rate at which energy is received is only 70 to 80 milliwatts per square centimeter.

The amount of energy radiated by different radiators depends on the character of the radiator and upon its temperature. For any radiator for which the intensity of the radiation for different wave lengths is known it is possible to calculate the percentage that will be transmitted by different thicknesses of water or body tissue using the data on the transmission of water given in Fig. 1, curves *A* and *B* or for body tissue using Carthwright's data as shown in Fig. 2. Unfortunately the character of radiation from all sources is not known. There are, however, a number of sources for which it is possible to calculate the energy radiated for different wave lengths and for different temperatures. The relation between the intensity of radiation for different wave lengths and different temperatures of the standard radiator, that is the black body, is known and thus the relation between the radiation intensity and wave lengths from the very short ultraviolet to the long infrared wave lengths for the black body can be calculated for any temperature desired. The radiation characteristics of tungsten have been determined for a wide range of temperatures and wave lengths so that it is possible to determine the relation between wave length and radiation intensity for tungsten for a wide range of temperatures. From results thus

obtained the percentage of the radiation from a tungsten lamp at different temperatures that is transmitted by water and by body tissue may be calculated.

Carbon filament lamps are sometimes used in these various infrared radiators. While there is not as much information available concerning the radiation characteristics of carbon as there is for tungsten the available data seem to show that carbon radiates sufficiently like a gray body so that results on the carbon lamp may be obtained by using the black body data. In some of these devices the radiation is from a black-

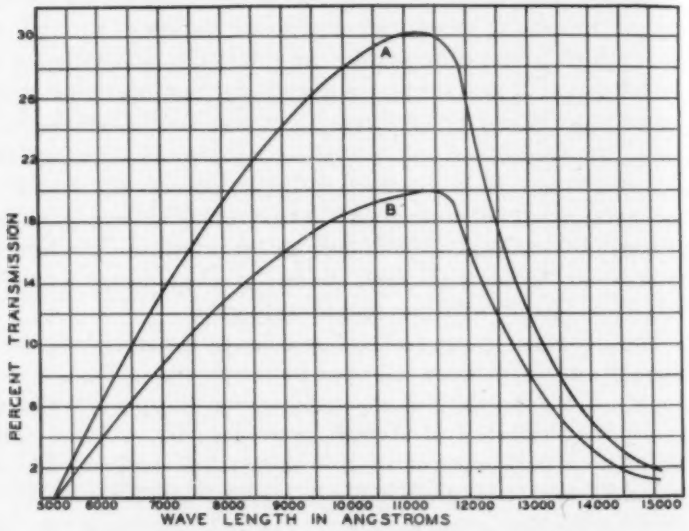

FIG. 2. *Carthwright's transmission curve of the human cheek for a thickness of five millimeters.*
Curve *B—Results obtained by Carthwright without considering reflection from the surface.*
Curve *A—Data in Curve B corrected for 34% reflection from the surface.*

ened iron heater which is heated to a temperature around 700 to 1000°K. It has been shown experimentally that oxidized iron radiates very much like a black body. Thus with a fair degree of accuracy the radiation from these blackened iron heaters can be determined by using the black body data.

Abbot and his co-workers at the Smithsonian Institution have determined both the intensity of the radiation from the sun outside of the earth's atmosphere for different wave lengths and the spectral trans-

mission of the atmosphere. Thus the spectral distribution of radiation intensity of the sun that reaches the earth's surface can be determined. From the data on these radiators, i.e., the iron heater at a temperature of 1000°K, the carbon lamp at 2150°K, the tungsten lamp at a temperature of 2970°K and Abbot's data on the sun's radiation the percentage of their radiation that is transmitted through various thicknesses of water and body tissue has been calculated and is shown in Table 1. Some substances transmit a varying amount of the radiation of all wave lengths received, while others transmit practically none in certain wave length regions and a large percentage of other regions. In such cases the material is said to have an absorption band or a transmission band as the case may be. The transmission of water may be said to consist of a number of bands, two of which are shown in Fig. 1, curves *A* and *B*. The first transmission band is rather wide while the second one is narrow. The amount of the total radiation transmitted by a sub-

TABLE 1. *Transmission of radiation from different sources through different thicknesses of water and body tissue.*

	Source of radiation			
	Iron heater 1000°K	Carbon lamp 2150°K	Tungsten lamp 2970°K	Sun
Flesh				
1 mm	0.58%	15.0%	30.0%	29.0%
1 cm	.02	.9	1.9	2.3
Water				
1 mm	3.1	35.	66.	85.
2 mm	1.4	28.	59.	81.
5 mm	.5	20.	51.	76.
1 cm	.25	15.	43.	71.
2 cm	.10	11.	36.	65.
1 cm Water and 1 mm Flesh			19.	
1 cm Water and 1 cm Flesh			1.2	

stance that has a transmission band is in some respects interesting. The data given in Table 1 show that one millimeter of water transmits 66% of the total radiation that it receives from a tungsten lamp at 2970° K. At first thought it might be said that the second millimeter would

transmit 66% of the amount transmitted by the first millimeter. Such is not the case at all. This is due to the fact that a very large amount of the energy reaching the first surface is not transmitted at all, while for the radiation of wave lengths corresponding to the transmission band a very large percentage that reached the first surface may pass through the first millimeter and a large percentage of this through the second millimeter. This point is well illustrated by the red glass screens that are used to produce monochromatic radiation for optical pyrometers. One such red glass screen placed in the eyepiece of the pyrometer will cut down the amount of light received from a black body at 2000°K to about 3% of what it was without any screen. A second red glass screen will not cut down the light to 3% of 3%, that is .09%, but the two screens will transmit about 1.8% of the light received. Thus the second screen transmits about 60% of the light transmitted by the first. A calculation has been made of the amount of light transmitted by the second millimeter and second centimeter for the different sources and different conditions and the results are also given in Table 1.

From the figures given in Table 1 it can be seen that a very large percentage of the energy transmitted by the first millimeter passes through the second millimeter. This is the explanation of the recommendation that is sometimes made that a water screen be used in front of a source of radiation if it is desired to have the radiation penetrate deeply into the body without undue discomfort to the patient. The radiation absorbed by the water cell is for the most part made up of those wave lengths that are absorbed in the outer layer of the body and it is the radiation that is absorbed in the outer layers of the body that is the cause of the uncomfortable heating by radiation.

Dr. E. Q. Adams, in a paper on the "Penetration of Radiation Into the Body Tissue,"[2] shows an interesting relation between the absorption and the depth of penetration. It is obvious that radiation must reach the desired depth and some of it must be absorbed at this depth if any value is to be obtained for radiation on the parts of the body at some distance below the surface. Little value will be obtained for conditions of very low or very high transmission, for in the first case very little or none of the radiation will reach the desired depth, while in the second case very little or none of the radiation will be absorbed to produce the desired effect. Dr. Adams shows that the most satisfactory results are to be obtained for such an absorption that 1/eth of the radiation reaches the desired depth.

[2] Physical Review, *36*, 1020; 1930.

In these calculations no account has been taken of the reflection of the surface of the water. The reflection would not affect the results except in absolute amount unless they are selective in favor of certain wave-length regions. Data given in the International Critical Tables for the spectral reflection of water show that the reflected factor amounts to 2 to 4% and that there is a very small variation in the reflecting factor with the wave lengths.

Dr. Sonne[3] reports that the reflection of the cheek for radiation from $\lambda = .5\mu$ to $\lambda = 1.5\mu$ is about constant and equal to about 34%. In the values given in Table 1 for the transmission of the body tissue a correction has been made for the 34% of the incident radiation that is reflected from the surface of the body. The figures thus given are for the percentage of the total radiation from the different sources that actually reaches the depths given. Some results are also given for the amount of energy that is transmitted by the body tissue after the radiation has passed through one centimeter layer of water. The results given for the amount of radiation from a heated iron radiator at a temperature of 1000°K show that about 3% of this energy is transmitted through one millimeter of water and only about .3% through one centimeter of water. Taking 34% as the amount of total energy reflected from the surface of the body for this wave-length region it is found that only about .6% of the energy from this heater is transmitted through one millimeter of body tissue and only about .02% through one centimeter of body tissue. In this case almost the entire amount of radiation that is not reflected from the surface is used in heating a very thin layer near the surface. Such a radiator would feel very hot to the patient. The radiation from the carbon lamp at 2150°K is much better in this respect since about 15% of the total energy radiated passes through one millimeter of the body tissue and about 1% of the total energy reaches the depth of one centimeter. The conditions are much better for the radiation from a tungsten lamp at high temperature as a source providing it is desired to have the radiation penetrate deep into the body tissue. For this case about 30% of the energy gets through the first millimeter and about 2% through the first centimeter. There is not much difference between the absorption by the body tissue of the radiation from the sun and the high temperature tungsten lamps. It is interesting to compare the amount of energy that reaches certain depths from each of these three sources, that is, the iron heater at 1000°K, the carbon lamp at 2150°K and a tungsten lamp at 2970°K for each radiating the same amount

[3] Dr. Sonne, Archives of Physical Therapy, X-ray & Radium, March, 1929.

of total energy. Using the low temperature heater only about .6% of the total energy gets through the first millimeter and only about .02% reaches a depth of one centimeter. For the carbon heater about 15% of the radiant energy gets through the first millimeter and about 1% through the first centimeter. Thirty percent of the energy radiated by the tungsten lamp gets through the first millimeter and about 2% through the first centimeter. A much larger percent of the energy from the low temperature radiator is either reflected from the surface or absorbed in the first millimeter than from a high temperature tungsten lamp. If a water screen one centimeter thick is placed between the tungsten lamp heater and the body surface only about 43% of the total radiation reaches the surface of the body and about 34% of this is reflected by the skin. Thus only about 29% of the total radiation enters the body and about 10% of the total is absorbed in the first millimeter while about 1.2% reaches a depth of one centimeter. Thus with the high temperature tungsten lamp as the source and the radiation first passed through a one centimeter water cell, for the same amount of total energy absorbed in the first millimeter, which gives the uncomfortable heating to the patient, about twice as much energy could be caused to reach the depth of one centimeter. These illustrations show very clearly the difference between a low temperature and a high temperature source for radiation therapy. If it is desired to have the energy for the most part absorbed in the outer layers a low temperature source should be used. On the other hand, if it is desired to have any appreciable amount of radiation penetrate very deep into the body tissue and at the same time avoid excessive surface heating, a high temperature source should be used.

THE FILLING OF A SPECTROGRAPH WITH LIGHT CONSIDERED AS A GEOMETRICAL RADIATION PROBLEM

By J. Rud Nielsen

[University of Oklahoma, Norman, Oklahoma. Received Aug. 13, 1930]

Introduction

The problem of determining the maximum amount of light that may be utilized in a given prism spectrograph in the case of a spatially extended light source has presented itself to the writer in an investigation on the Raman effect. It is of considerable importance whenever a radiation of low intensity is the object of spectroscopic study. In the present paper this problem will be discussed from the point of view of geometrical optics, i.e. without taking diffraction into account. It is assumed, to begin with, that no condensing lens is used in front of the spectrograph slit. This assumption is dropped in the second part of the paper, and the most efficient use of a condensing lens is investigated. Finally, a brief discussion is given, in the light of the formulas developed, of the speed of a spectrograph and the design of the collimator.

Spectrograph Used Without Condensing Lens

Let the slit, which will be assumed vertical for the sake of definiteness, have the length $2s$ and the very small width w. Let the collimator lens be large enough to fill the entire prism, and let $2a$ denote the width of the parallel beam of light which enters the prism and $2b$ the height of the prism. In order to simplify the discussion, we shall assume that a diaphragm with an aperture with sides $2a$ and $2b$ is placed in front of the collimator lens. In order to distinguish between the various parts of the luminous source, we make use of a coordinate system with its origin in the midpoint of the slit, with the z-axis coinciding with the collimator axis, and with the x-axis perpendicular and the y-axis parallel to the slit.

Consider an infinitesimal length $d\sigma$ of the slit, of distance σ from the origin. The amount of light entering this part of the slit in unit time and coming from the volume element $dxdydz$ of the source at x, y, z is readily seen to be

$$\frac{1}{4\pi} I(x, y, z) w (x^2 + (y - \sigma)^2 + z^2)^{-3/2} z dx dy dz d\sigma, \qquad (1)$$

where $I(x, y, z)$ is the amount of radiation emitted equally in all directions in unit time by unit volume of the source at x, y, z. It is supposed that no absorption occurs between the point x, y, z and the slit.

Neglecting the diffraction in the slit, it is seen that the light, coming from the volume element $dxdydz$ and passing through the slit, will not enter the aperture of the collimator diaphragm unless the point x, y, z lies between two vertical planes U and U', Fig. 1a, which pass through the slit and through the vertical sides of the aperture; i.e. unless $-az/f \leqq x \leqq az/f$.

In Fig. 1b are indicated four additional planes V, V', W and W' which pass through the horizontal sides of the collimator aperture and the end points of the slit. Since, for any spectrograph, $2s<2b$, the planes V and V' will cut one another in a horizontal line lying at a distance $z_0=fs/(b-s)$ in front of the slit. The region bounded by the planes U, U', V V' and containing the z-axis will be called the "inner region," while the region bounded by the planes V, V', W, W' as well as by the

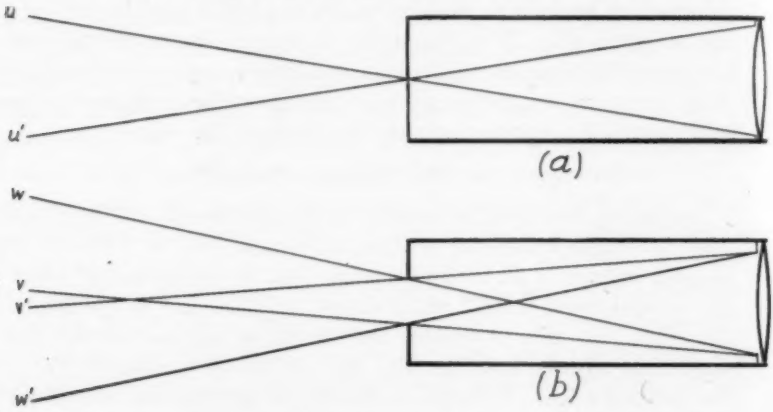

FIG. 1. *Diagram of spectrograph collimator* (a) *Horizontal section.* (b) *Vertical section.*

vertical planes U, U', will be called the "outer region." If the source point x, y, z lies in the inner region, and if $z>z_0$, all the light entering the slit will also enter the collimator aperture. On the other hand, when $0<z<z_0$, the light passing through the elment $d\sigma$ of the slit will reach the collimator lens only if $(fy-bz)/(f+z) \leqq \sigma \leqq (fy+bz)/(f+z)$. If the point x, y, z lies in the outer region, a part of the light entering the slit will always miss the collimator aperture. The condition that the light passing through the element $d\sigma$ of the slit reaches the aperture, is $(fy-bz)/(f+z) \leqq \sigma \leqq s$ when $y>0$ and $-s \leqq \sigma \leqq (fy+bz)/(f+z)$ when $y<0$. A luminous point outside the outer region can send no light into the collimator aperture.

The total amount of light sent in unit time from the volume element $dxdydz$ into the collimator aperture is obtained by integrating (1) with respect to σ, using the appropriate limits depending on the region in which x, y, z is located. We get for each region an expression of the form

$$\frac{1}{4\pi} I(x, y, z) w \Phi(x, y, z) dx dy dz. \tag{2}$$

In order to distinguish between the various functions Φ, we make use of superscripts i and o to indicate the inner and outer regions respectively and subscripts *1* and *2* to indicate whether z is smaller or greater than z_0. The functions obtained are as follows:

$$\left.\begin{aligned}
\Phi_1^{\,i}(x, y, z) &= \frac{z}{x^2 + z^2} \cdot \left(\frac{(b - y)z}{\sqrt{(b - y)^2 z^2 + (f + z)^2 (x^2 + z^2)}} \right. \\
&\qquad \left. + \frac{(b + y)z}{\sqrt{(b + y)^2 z^2 + (f + z)^2 (x^2 + z^2)}} \right), \\
\Phi_2^{\,i}(x, y, z) &= \frac{z}{x^2 + z^2} \cdot \left(\frac{s - y}{\sqrt{(s - y)^2 + x^2 + z^2}} \right. \\
&\qquad \left. + \frac{s + y}{\sqrt{(s + y)^2 + x^2 + z^2}} \right), \\
\Phi_1^{\,o}(x, y, z) &= \frac{z}{x^2 + z^2} \cdot \left(\frac{s - |y|}{\sqrt{(s - |y|)^2 + x^2 + z^2}} \right. \\
&\qquad \left. + \frac{(b + |y|)z}{\sqrt{(b + |y|)^2 z^2 + (f + z)^2 (x^2 + z^2)}} \right), \\
\Phi_2^{\,o}(x, y, z) &= \Phi_1^{\,o}(x, y, z).
\end{aligned}\right\} \tag{3}$$

The integration of (2) with respect to x, y, and z over the volume of the source will, in general, be extremely difficult to carry out. We shall consider only the case that $I(x, y, z)$ is a constant, independent of x, y, z, and shall assume further that the luminous source has a cross section large enough to embrace everywhere the entire outer and inner regions. These conditions are often satisfied, at least approximately, in experiments on the Raman effect.

Assuming that I is a constant, we integrate (2) with respect to x and

y first over the cross section of the inner region in the case that $z \leqq z_0$, next over the cross section of the inner region when $z \geqq z_0$; then over the cross section of the outer region in case $z \leqq z_0$, and finally over the cross section of the outer region for values of $z \geqq z_0$. The limits of integration are found by simple geometrical considerations. The result is in each case of the form

$$\frac{1}{\pi} w I \Psi(z) dz.$$

Using again the superscripts i, o and the subscripts 1, 2 to distinguish between the various cross sections, we find

$$\left.\begin{aligned}
\Psi_1{}^i(z) &= \left(b + s - \frac{(b-s)z}{f}\right) \tan^{-1} \frac{b + s - \dfrac{(b-s)z}{f}}{P(z)} \\
&\quad - \frac{1}{2}(f+z) \log \frac{P(z) - f - z}{P(z) + f + z} - A(f+z), \\
\Psi_2{}^i(z) &= -\left(2s - \frac{(b-s)z}{f}\right) \tan^{-1} \frac{2s - \dfrac{(b-s)z}{f}}{Q(z)} \\
&\quad + \frac{1}{2} z \log \frac{Q(z) - z}{Q(z) + z} + Az, \\
\Psi_1{}^o(z) &= -\left(b + s - \frac{(b-s)z}{f}\right) \tan^{-1} \frac{b + s - \dfrac{(b-s)z}{f}}{P(z)} \\
&\quad + \frac{1}{2}(f+z) \log \frac{P(z) - f - z}{P(z) + f + z} + Az + Bf, \\
\Psi_2{}^o(z) &= \left(2s - \frac{(b-s)z}{f}\right) \tan^{-1} \frac{2s - \dfrac{(b-s)z}{f}}{Q(z)} \\
&\quad - \frac{1}{2} z \log \frac{Q(z) - z}{Q(z) + z} - A(f+z) + Bf,
\end{aligned}\right\} \quad (5)$$

where the following abbreviations are used

$$
\left.
\begin{aligned}
P(z) &= \frac{1}{a}[(f^2 + a^2 + (b-s)^2)z^2 + 2(f^2 + a^2 - b^2 + s^2)z \\
&\quad + (f^2 + a^2 + (b+s)^2)f^2]^{1/2}, \\
Q(z) &= \frac{1}{a}[(f^2 + a^2 + (b-s)^2)z^2 - 4sf(b-s)z + 4s^2f^2]^{1/2}, \\
A &= \frac{b-s}{f}\tan^{-1}\frac{a(b-s)}{f\sqrt{f^2 + a^2 + (b-s)^2}} \\
&\quad - \frac{1}{2}\log\frac{\sqrt{f^2 + a^2 + (b-s)^2} - a}{\sqrt{f^2 + a^2 + (b-s)^2} + a}, \\
B &= \frac{b+s}{f}\tan^{-1}\frac{a(b+s)}{f\sqrt{f^2 + a^2 + (b+s)^2}} \\
&\quad - \frac{1}{2}\log\frac{\sqrt{f^2 + a^2 + (b+s)^2} - a}{\sqrt{f^2 + a^2 + (b+s)^2} + a}.
\end{aligned}
\right\} \quad (6)
$$

It will be noticed that

$$\Psi_1^i(z) + \Psi_1^o(z) = \Psi_2^i(z) + \Psi^o(z) = (B - A)f. \quad (7)$$

Thus, if the cross section of the source is everywhere wide enough to include the entire inner and outer regions, and if the source is bounded by the planes $z = z_1$ and $z = z_2$ and has a uniform brightness I, the total amount of light sent into the collimator aperture will be

$$\frac{1}{\pi} wIC(z_2 - z_1) \quad (8)$$

where

$$C = (B - A)f, \quad (9)$$

Hence, the amount of light entering the collimator aperture is, in this case, proportional to the longitudinal extension or depth $z_2 - z_1$ of the light source and is independent of its distance from the slit. Although this interesting result hardly can be new, the writer has found no mention of it in the literature.

Numerical example.

Let us apply formulas (3) and (5) to the case of a Hilger *E1* quartz spectrograph. For this instrument $f = 155$ cm (for 2537A), $a = b = 2.55$ cm, and $s = 0.14$ cm (when a particular Hartmann diaphragm is used in front of the slit); hence $z_0 = 9.0$ cm. For a given z, the horizontal extension of either the inner or the outer region is determined by $2x_0 =$

$2az/f$. The vertical width of the inner region is given by $2y_0=2|s-(b-s)z/f|$, while the distance from the axis to the outer edge of the outer region is $y_1=s+(b+s)z/f$. In Table 1 are listed the values of x_0, y_0, and y_1 for three different values of z.

TABLE 1.

z	x_0	y_0	y_1
6 cm	0.099 cm	0.047 cm	0.244 cm
9	0.148	0.000	0.296
12	0.197	0.047	0.348

First consider the variation of the functions $\Phi(x, y, z)$ with x when y and z are kept constant. We find, for instance, from (3) after expansion

$$\Phi_1{}^i(x, 0, 6) = 0.0526(1 - 0.0413x^2).$$

The variation of this function as x varies from 0 to $x_0=0.099$ cm is thus less than 0.05%, i.e., $\Phi_1{}^i(x, 0, 6)$ is practically a constant. The same result is found for other values of y and z. For the other functions $\Phi_2{}^i(x, y, z)$ and $\Phi_1{}^o(x, y, z)=\Phi_2{}^o(x, y, z)$ we similarly find that, within the range $-x_0<x<x_0$, they are practically independent of x.

Next consider the variation of the Φ's with respect to y. We find, after expanding the various terms, that

$$\Phi_1{}^i(0, y, 6) = 5.27 \times 10^{-3} - 3.05 \times 10^{-7}y^2.$$

For $|y| \leqq y_0=0.047$ cm this expression is practically constant. The same is found to be true for other values of x and z. Similarly, the function $\Phi_2{}^i(x, y, z)$ is practically independent of y for $|y| \leqq y_0$. For the functions $\Phi_1{}^o(0, y, 6)$ and $\Phi_2{}^o(0, y, 12)$ we find on the other hand

$$\Phi_1{}^o(0, y, 6) = 6.53 \times 10^{-3} - 2.67 \times 10^{-2}y - 1.61 \times 10^{-4}y^2,$$

$$\Phi_2{}^o(0, y, 12) = 2.25 \times 10^{-3} - 6.54 \times 10^{-3}y - 1.01 \times 10^{-5}y^2.$$

These functions decrease practically linearly from a maximum value to zero as y increases from y_0 to y_1. Summarizing all these results, we conclude that, with a spectrograph of as low a speed as that of the Hilger *E*1, all source points lying within the inner region and having a given coordinate z are essentially equally effective in sending light into the spectrograph. On the other hand, for a volume element $dxdydz$ lying in the outer region, the amount of light sent into the collimator lens decreases nearly linearly as the volume element moves from the inner to the outer boundary of this region.

We next consider the functions Ψ_1^i, Ψ_2^i, Ψ_1^o, and Ψ_2^o defined by equations (5) and (6). Since in the case of the Hilger *E1* spectrograph, f is large compared with a, b, and s, the various terms in (5) may be expanded. When that is done, the following approximate expressions are obtained:

$$\left.\begin{aligned} \Psi_1^i(z) &= K\left(1 - \frac{bz}{s(f+z)}\right), \\ \Psi_2^i(z) &= K\left(1 - \frac{s(f+z)}{bz}\right), \\ \Psi_1^0(z) &= K\frac{bz}{s(f+z)}, \\ \Psi_2^0(z) &= K\frac{s(f+z)}{bz}, \end{aligned}\right\} \tag{11}$$

where

$$K = \frac{2abs}{f^2}. \tag{12}$$

Fig. 2 shows the variation of these functions with z. It will be noticed that at distances from the slit of between ca. 4.5 cm and ca. 19.5 cm the largest amount of light comes from the outer region, while the inner region contributes more light for values of z smaller than 4.5 cm or larger

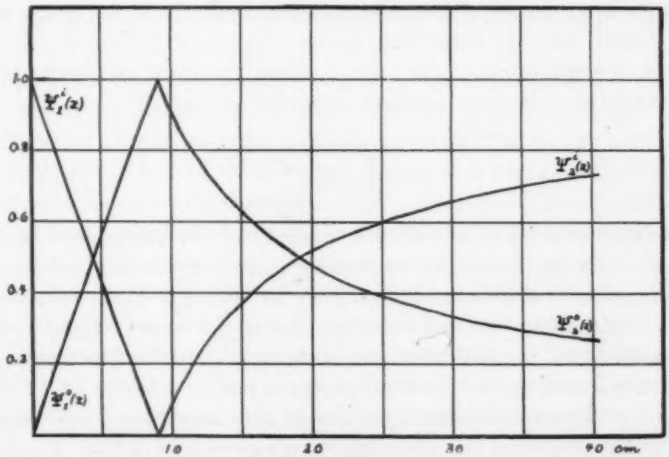

FIG. 2. *Graph of the functions $\psi(z)/K$ for a Hilger E 1 quartz spectrograph.*

than 19.5 cm. It should be remembered that it is assumed here that the light source has a uniform brightness I and a cross section large enough to embrace the entire inner and outer regions. From (11) and (12) it follows that

$$\Psi_1^i(z) + \Psi_1^0(z) = \Psi_2^i(z) + \Psi_2^o(z) = \frac{2abs}{f^2}. \tag{13}$$

The right hand term of this equation can also be obtained by expansion of the constant C defined by (9).

MOST EFFICIENT USE OF CONDENSING LENS

Now let a condensing lens of focal length F be placed between the source of radiation and the spectrograph slit. If the aperture of the condensing lens is large enough to cover the entire cross section of the space bounded by the planes U, U', W, W', and if the lateral extension of the source is sufficiently great, then the amount of light sent into the spectrograph will be exactly the same as when no condensing lens is used. This fact is well known. However, since it is usually proved only for a luminous surface as source, the general proof will be briefly indicated.

If the distance from the condensing lens to a volume element $dxdydz$ of the source is z, and the distance to the image of this volume element ζ, then it follows from the simple lens formula that $d\zeta = -(F/(z-F))^2 dz$. By comparing the magnitude of an infinitesimal solid angle $d\Omega$, with apex at x, y, z, with the conjugate solid angle $d\Omega'$ in the image space, it is found that the brightness of the image is $I' = I((z-F)/F)^2$ where I is the brightness of the source. Now, the total amount of light entering the collimator aperture is, by (4), (7) and (9),

$$\int_{\zeta_1}^{\zeta_2} \frac{1}{\pi} w I' C d\zeta.$$

Substituting for I' and $d\zeta$ and integrating, the expression (8), derived above for the case of no condensing lens, is again obtained. It should be noticed that this result is entirely independent to the focal length of the condensing lens and of which part of the source, if any, is focused on the slit. In particular, it is immaterial whether the image formed of the source by the condensing lens is real or virtual.

In the case of a radiating source of very large lateral extension, it is, therefore, never of any advantage to use a condensing lens. On the other hand, when the source has a limited cross section, the use of a conden-

sing lens is often of the utmost importance. In the first place, if both the longitudinal and the lateral extensions of the light source are small, and if it is impossible to place the source very close to the spectrograph slit, an image of the source may be formed on the slit by means of a suitable condensing lens, so as to make the amount of light sent into the spectrograph the same as if the source were placed directly in front of the slit. In the second place, when the source of radiation has a large longitudinal extension, as in the case of the so-called "end-on" illumination, it will often be possible, by means of a condensing lens, to increase greatly the amount of light sent into the collimator aperture of the spectrograph. This is of particular importance in work on the molecular scattering of light and on the Raman effect. Except for a few lines in Kayser's Handbuch der Spektroskopie,[1] the writer has found no discussion of the latter use of a condensing lens in the literature. A rather complete analysis will, therefore, be given here.

In view of (8), the problem may be formulated as follows. Let the luminous source have a given uniform cross section, which for simplicity will be assumed rectangular, with horizontal sides $2g$ and vertical sides $2h$. For a given spectrograph and a given condensing lens, what is then the greatest length l_m of the source for which it is possible to fill the spectrograph with the maximum amount of light as given by (8)? This maximum length l_m of the source will, of course, depend upon the focal length F of the condensing lens; hence we must further determine the greatest possible value l_{max} of l_m obtained when F has the most favorable value.

The condition that a luminous source of given cross section $2g \times 2h$ and length l fills a spectrograph with the maximum amount of light is that the part of the spatial image of the source which lies in front of the spectrograph slit or behind the collimator aperture is everywhere wider than the space bounded by the planes U, U', W, and W', and that the part of the image which lies between the slit and the collimator aperture is wider than the space bounded by the planes U, U', V, and V'. We shall first consider the conditions involving the planes V, V', W, and W'. The notations s, a, b, f and z_0 used above will be used again. It will be convenient also to make use of the symbol z_1 for the distance from the slit to the point where the planes W and W' cut the collimator axis. It is readily seen that $z_1 = fs/(b+s)$; and it will be remembered that $z_0 = fs/(b-s)$. We shall assume that g and h are not very different and

[1] H. Kayser, Handbuch der Spektroskopie, Vol. I, p. 581.

that h is considerably larger than s. It is necessary to divide the discussion into five parts according to the value F of the focal length of the condensing lens.

Case I: $$F \leq \frac{(h-s)h}{(h+s)(b+s)}f.$$

This case is shown in Fig. 3. H and H' indicate the boundaries of the light source. We assume that the source is infinitely long and extends clear to the condensing lens L. Our problem is then to determine the length l of the source which is fully efficient in sending light into the spectrograph. The image of any point on the boundaries H and H' of the light source will lie on one of the planes X or X'. The positions of the spectro-

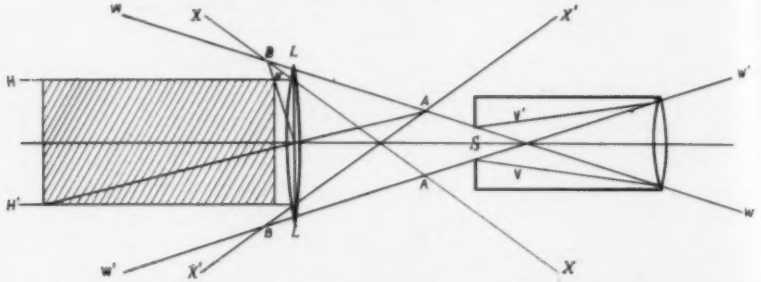

FIG. 3. *Graphical determination of the length of the source which is fully efficient in sending light into the spectrograph. HH': horizontal boundaries of source. L: condensing lens. S: slit of spectrograph.*

graph slit S and of the planes WW', V and V' are indicated to the right in the figure. It is clear that only the cross sections of the image of the source which lie to the right of the points A or to the left of the points B will be fully efficient in filling the spectrograph. Hence, the effective part of the source (shaded in the figure) lies between the points A' and B' which are conjugate to A and B.

If z denotes the distance from the slit to the condensing lens, then

$$l = \frac{2hz_1F}{s(z+z_1-F)} \tag{14}$$

by the simple lens formula. The length l obviously increases as z decreases, *i.e.*, as the spectrograph is moved closer to the condensing lens. However, this expression for l holds only as long as z is sufficiently large. If the spectrograph is moved so close to the condensing lens that z

becomes smaller than $(h/s-1)z_1$, then B will move to the right of L. The fully efficient part of the source will now extend clear to L and we get the new expression

$$l = \frac{(z + (h/s + 1)z_1)F}{z + z_1 - F}. \tag{15}$$

This expression also increases as z is decreased. It holds as long as the plane W rather than the plane V' cuts X', *i.e.*, as long as $z \geqq (h+s)F/h$. As z decreases below this value, A will move to the right along X' and l will decrease. Hence the largest value of l is

$$l_m^{\text{I}} = \frac{h+s}{s}F \quad \text{for} \quad z = \frac{h+s}{h}F. \tag{16}$$

This expression has its greatest value l_{max} when the upper limit for F is substituted.
Hence,

$$l_{\max}^{\text{I}} = \frac{(h-s)h}{(b+s)s}f, \qquad F = \frac{h-s}{h+s}\,\frac{h}{b+s}f. \tag{17}$$

It is interesting to compare this length with the length l_0 of the source which may be fully utilized when no condensing lens is used. It is readily found that

$$l_0 = \frac{h-s}{b+s}f. \tag{18}$$

Hence

$$\frac{l_{\max}^{\text{I}}}{l_0} = \frac{h}{s}. \tag{19}$$

Case II: $$\frac{h-s}{h+s}\,\frac{h}{b+s}f \leqq F \leqq \frac{h}{b+s}f.$$

In this case B will remain to the left of L for all values of z greater than $(h+s)F/h$ (and, in general, for somewhat smaller values). Hence, for $z \geqq (h+s)F/h$ the efficient length of the luminous source is given by (14). This expression increases with decreasing z and has the maximum value

$$l_m^{\text{II}} = \frac{2h^2Ff}{s[(b+s)F + hf]} \quad \text{for} \quad z = \frac{h+s}{h}F. \tag{20}$$

The greatest possible value l_{max}^{II} of l_m^{II} is obtained when F is given its greatest permitted value:

$$l_{max}^{II} = \frac{h^2}{s(b+s)}f; \quad F = \frac{h}{b+s}f. \tag{21}$$

Hence

$$\frac{l_{max}^{II}}{l_0} = \frac{h^2}{s(h-s)}. \tag{22}$$

We must now investigate if it is possible to increase l by moving the spectrograph closer to the condensing lens. When z is reduced below $(h+s)F/h$, A will be the point of intersection of X' and V', rather than of X' and W, and, as long as B remains to the left of L, *i.e.*, as long as $z>(h-s)f/(b+s)$, we have for the efficient length

$$l = \left(\frac{z - \left(\frac{h}{s}+1\right)z_0}{z - z_0 - F} - \frac{z - \left(\frac{h}{s}-1\right)z_1}{z + z_1 - F} \right) F. \tag{23}$$

This expression has a minimum at $z=z'$ and a maximum at $z=z''$, where z' is the larger and z'' the smaller of the values

$$\left.\begin{matrix} z' \\ z'' \end{matrix}\right\} = \left(1+\frac{b}{h}\right)F - f \pm \frac{b}{h}\sqrt{\left(\frac{hf}{b-s}-F\right)\left(\frac{hf}{b+s}-F\right)}. \tag{24}$$

The quantity $(1+b/h)F-f$ has as its greatest possible value $(h-s)f/(b+s)$, which is reached when $F=hf/(b+s)$. Hence z'' will never lie in the interval $[(h-s)f/(b+s),(h+s)F/h]$. z' may or may not lie in this interval. In either case, since $z=z'$ corresponds to a minimum of l, it will be sufficient to evaluate l for the lower limit of the interval and compare the expression obtained with (20). We find

$$l = \frac{[(h+s)(b+s)-(h-s)(b-s)]fF}{(b^2-s^2)F - [(h-s)(b-s)-s(b+s)]f} \quad \text{for } z = \frac{h-s}{b+s}f \tag{25}$$

It can be shown that this expression is never larger than l_m^{II} as given by (20), provided $h>bs/b-s$. As we have already assumed that h is considerably larger than s, this condition will certainly hold.

It may be seen without any analysis, as in Case I, that l will steadily decrease, when z is reduced below the value $(h-s)f/(b+s)$. (20) and(21) thus give the correct maximum values for Case II.

Case III: $$\frac{h}{b+s}f \leqq F \leqq \frac{h}{b-s}f.$$

This case is as simple as Case I. Only the results will be given. l_m is reached when $z=(h+s)F/h$, and is given by (20), while

$$l_{\max}^{\text{III}} = \frac{h^2}{sb}f; \quad F = \frac{h}{b-s}f \tag{26}$$

and

$$\frac{l_{\max}^{\text{III}}}{l_0} = \frac{h^2(b+s)}{(h-s)sb}. \tag{27}$$

Case IV: $$\frac{h}{b-s}f \leqq F \leqq \frac{hb}{(b-2s)(b+s)}f.$$

When $z \geqq (1+b/h)F-f$ the effective length of the source is again given by (14). It increases as z decreases, reaching the value

$$l_m^{\text{IV}} = \frac{2h^2z_1F}{s[bF-h(f-z_1)]} \tag{28}$$

when $z=(1+b/h)F-f$.

This expression increases with decreasing F. Its maximum value, reached when $F=hf/(b-s)$, is given by (26). In order to be sure that (28) and (26) actually give l_m^{IV} and l_{max}^{IV}, we must consider the values of l obtained when $z \leqq (1+b/h)F-f$. It may be readily seen from a drawing that $l=0$ when $z \leqq F(h+s)/h$, so we are concerned only with the interval

$$[(h+s)F/h,\ (1+b/h)F-f].$$

In this interval we have

$$l = \left(\frac{z+\left(\frac{h}{s}+1\right)z_1}{z+z_1-F} - \frac{z-\left(\frac{h}{s}+1\right)z_0}{z-z_0-F} \right) F. \tag{29}$$

This expression has a maximum at $z=z'$ and a minimum at $z=z''$, where z' is the larger and z'' the smaller of the values

$$\left.\begin{matrix} z' \\ z'' \end{matrix}\right\} = \left(1+\frac{s}{h}\right)F \pm \frac{s}{h}\sqrt{\left(F-\frac{hf}{b-s}\right)\left(F+\frac{hf}{b+s}\right)}.$$

The value z'' will never lie in the interval considered. The condition that z' lies in the interval is found to be

$$F > \frac{hb}{(b-2s)(b+s)}f,$$

so, for the range of values of F that we consider in Case IV, (28) and (26) are valid.

Case V:

$$F \geqq \frac{hb}{(b-2s)(b+s)}f$$

In this case z' will lie in the interval considered under Case IV, and l_m will be obtained by substituting

$$z' = \left(1+\frac{s}{h}\right)F + \frac{s}{h}\sqrt{\left(F-\frac{hf}{b-s}\right)\left(F+\frac{hf}{b+s}\right)} \tag{31}$$

in (29). We find

$$l_m^{\mathrm{V}} = \frac{2h^2bfF}{s(b^2-s^2)\left(\sqrt{F-\frac{hf}{b-s}}+\sqrt{F+\frac{hf}{b+s}}\right)^2}. \tag{32}$$

This expression decreases with increasing F. Hence,

$$l_{\max}^{\mathrm{V}} = \frac{h^2}{s(b+s)}f, \quad F = \frac{hb}{(b-2s)(b+s)}f \tag{33}$$

and

$$\frac{l_{\max}^{\mathrm{V}}}{l_0} = \frac{h^2}{s(h-s)}. \tag{34}$$

This completes our problem as far as the vertical extensions of the luminous source and of the spectrograph slit and collimator aperture are involved.

The restriction of the effective length of the source due to its width $2g$ is determined by a set of formulas identical with those just derived, except that s is replaced by $\frac{1}{2}w$ and b by a. However, a closer examination of these formulas is unnecessary, for it may be readily seen that if w is very small compared with s, and a and g not greatly different from b and h respectively, then the fully efficient part of the source will always be determined by the formulas (14) to (34).

The application of these formulas is obvious. It will be seen that the values of l_m first increase and then decrease with increasing focal length of the condensing lens. $F=hf/(b-s)$ is the most favorable value of the focal length. For this value of F it is possible to send into the collimator aperture the full amount of light from a luminous source of length h^2f/sb. This is $h^2(b+s)/s(h-s)b$ times the length of the longest tube which can be fully utilized when no condensing lens is used. The distance from the spectrograph slit to the condensing lens must be $(h+s)f/(b-s)$, and the condensing lens must be large enough to embrace completely a rectangle with sides $a(h+s)/(b-s)$ and $[(b+s)(h+s)/(b-s)]+s$. The axis of the source must coincide with the axis of the collimator, and the back surface of the source must be focused on the slit. Under these circumstances, no point of the walls of the source will send any light into the collimator of the spectrograph.

In his work on the Raman effect in hydrogen chloride, Wood[2] focused the front surface of the source on the spectrograph in order to prevent as much as possible of the light diffracted in the aperture at the front of the source from reaching the slit. It would seem that it had been better to avoid a narrow aperture at the front surface of the source and thus make possible the use of a much longer source.

In deriving the formulas given above, the condensing lens has been considered as a thin lens obeying the simple lens formula. It is, therefore, necessary to exercise a certain amount of judgment as to how accurately the formulas can be expected to hold in a given practical case, especially when s is very small compared with h and when the spectrograph is fast.

It has been assumed that the index of refraction of the luminous space is very nearly equal to that of air. However, it is not difficult to generalize the formulas so that they will hold for any index of refraction of the source.

SPEED OF A SPECTROGRAPH AND DESIGN OF THE COLLIMATOR

There exists no generally adopted definition of the speed of a spectrograph, and many confusing statements on this subject are found in the literature.[3] Undoubtedly this is due to the fact that the factors deter-

[2] R. W. Wood, Phil. Mag. 7, 748; 1929.

[3] As an example may be mentioned the following statement by E. v. Angerer in his otherwise excellent article in Wien-Harms, Handbuch der Experimentalphysik, Vol. 21, p. 233: "Die Lichtstärke eines Spektrographen ist gleich der des Objektives mit dem kleinsten Öffnungsverhältnis."

mining the speed or sensitivity differ greatly in the cases of visual, bolometric, and photographic observation. It is well known that the brightness of a spectral line, in the case of visual observation, is entirely independent of the focal lengths of the collimator and telescope objectives, provided the pupil of the eye is completely filled with light. The cases of bolometric and photographic observation will now be briefly discussed in the light of the expression (8) given above.

This expression represents the maximum amount of light which can be gathered by a collimator from a source of uniform brightness and given longitudinal extension. The factor C depends entirely upon the dimensions of the collimator. It seems reasonable, therefore, to base the definition of the speed of a collimator upon this quantity. In the case of bolometric observation, the deflection obtained when the collimator is completely filled with light is proportional to C. The focal length of the lens which focuses the light on the bolometer strip (or thermo-couple junction), or on the exit slit, is essentially of no importance. Thus C may be taken as a measure of the speed or sensitivity in this case.

To obtain an expression for the speed in case of photographic observation, we differentiate C partially with respect to s. This gives

$$\frac{\partial C}{\partial s} = \tan^{-1}\frac{a(b+s)}{f\sqrt{f^2+a^2+b+s)^2}} + \tan^{-1}\frac{a(b-s)}{f\sqrt{f^2+a^2+(b-s)^2}}. \quad (35)$$

Considering s as a variable, (35) may be used for computing the variation in the intensity from the midpoint to the endpoint of a spectral line. The intensity at the midpoint of the line will be proportional to

$$S = \left(\frac{f}{f_1}\right)^2\left(\frac{\partial C}{\partial s}\right)_{s=0} = 2\left(\frac{f}{f_1}\right)^2\tan^{-1}\frac{ab}{f\sqrt{f^2+a^2+b^2}} \quad (36)$$

where f_1 is the focal length of the camera lens. The factor $(f/f_1)^2$ represents the ratio of the area of the slit aperture to the area of the spectral line upon the photographic plate. S may be taken as a measure of the speed of a spectrograph.

Since f is generally large compared with a, b, and s, (35) and (36) may be simplified by expansion, thus giving

$$\frac{\partial C}{\partial s} = \frac{2ab}{f^2} - \frac{ab}{f^4}(a^2+b^2+3s^2) \quad (35a)$$

and

$$S = \frac{2ab}{f_1^2} - \frac{ab}{f^2 f_1^2}(a^2 + b^2). \tag{36a}$$

The last term in (36a) can usually be disregarded. Thus, the speed of a spectrograph is practically independent of the focal length of the collimator lens and is proportional to the area of the collimator aperture and inversely proportional to the square of the focal length of the camera lens.

It should be emphasized that (36) gives what might be termed the "geometrical speed" of a spectrograph. The actual speed will always be lower due to diffraction, reflection and absorption of the light as well as to defects of the lenses and prisms and imperfect filling of the spectrograph with light. It should also be noted that, with a camera lens of very short focal length and with a narrow slit, the width of the spectral lines will be determined largely by the fineness of grain of the photographic plate. Under these circumstances the speed (in so far as this term has a meaning) will be obtained by replacing $(f/f_1)^2$ with f/f_1 in (36).

Now, let us once more consider a source of cross section $2g \times 2h$, and let us investigate how the length of the source which may be fully utilized depends on the focal length f of the collimator lens, if the focal length f_1 of the camera lens is given, and if it is required that the spectral lines have the length $2s_1$.

If no condensing lens is used, the greatest length which can be fully utilized is given by (18). Since $s = s_1 f/f_1$

$$l_0 = \frac{hf_1 - s_1 f}{bf_1 + s_1 f} f \tag{37}$$

For small values of f this expression increases with increasing f. It has the maximum value

$$(l_0)_{\max} = \frac{bf_1}{s_1}\left(\sqrt{1 + \frac{h}{b}} - 1\right)^2 \tag{38}$$

when

$$f = \frac{bf_1}{s_1}\left(\sqrt{1 + \frac{h}{b}} - 1\right).$$

If next a condensing lens of the most favorable focal length is used, the

maximum efficient length of the source is given by (26). Since $s = s_1 f / f_1$ we get

$$l_{\max}^{\mathrm{III}} = l_{\max}^{\mathrm{IV}} = \frac{h^2}{s_1 b} f_1. \tag{39}$$

This expression is entirely independent of f and is always greater than $(l_0)_{\max}$.

Thus we find again that the focal length of the collimator lens of a spectrograph is of no essential importance. When the focal length is rather long, it may often be possible to avoid the use of a condensing lens. When a condensing lens of the most favorable focal length is used, the value of the focal length of the collimator lens is entirely immaterial. However, the shorter the focal length of the collimator lens, the smaller will be the focal length and the diameter, and hence, within certain limits, the cost, of the condensing lens required.

PHYSICAL CHARACTERISTICS OF AC THERAPEUTIC CARBON ARCS

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Received Aug. 25, 1930]

Introduction

Numerous studies have been published of the operating characteristics of certain types of therapeutic arcs and of the relation between their output of ultraviolet radiation and their operating characteristics. Little has appeared however on the therapeutic carbon arc. Coblentz, Dorcas and Hughes[1] in their radiometric measurements on ultraviolet light sources did not consider especially the effect of varying conditions on the output. Griffith, Taylor and Jack[2] have studied the operating characteristics of direct current arcs between solid carbons, and measured the ultraviolet output with a quartz cadmium cell, which is sensitive to the ultraviolet below about 3200 AU. Thorsen[3] worked with neutral cored carbons and measured the ultraviolet energy in each of the three CN bands at λ4216, 3880 and 3590 by the use of a thermocouple which was moved along the focal plane of a quartz spectrograph. As far as we are aware, however, nothing has been published on the therapeutic cored carbons which are in general use in this country. Such a study would appear to be of interest in view of the large variety of arc lamps now sold for therapeutic use which use these carbons.

The purpose of this study has been to obtain data which would allow the plotting of the characteristic curves of several of the various sizes and types of therapeutic carbons for the operating ranges of current and voltage met with in practice. At the same time it seemed desirable to make observations on the ultraviolet output of the various arcs so as to determine the connection between current, voltage, arc length and ultraviolet output.

The fundamental characteristics of the electric arc between carbon electrodes have long been recognized. In fact Edlund[4] as early as 1867 observed that in a direct current arc between solid carbons at a constant current the apparent resistance is equal to a constant resistance

[1] Coblentz, Dorcas and Hughes, Sci. Papers, B. of S. *21*, 535; 1926.
[2] Griffith, Taylor and Jack, British Journal of Radiology *23*, 203; 1927.
[3] Thorsen, Strahlentherapie *34*, 46; 1929.
[4] Edlund, Pogg. Ann. *131*, 586; 1867.

plus a resistance which varies directly with the length of the arc, or in equation form $R=a+bL$, where $R=$ resistance, $L=$ length of arc and a and b are constants. If this equation be multiplied by the current we have the fundamental equation for arc voltage, $V=V_0+bL$. This relation has been shown to hold also for alternating current arcs by Heubach[5] for both homogeneous and neutral cored carbons (Dochtkohlen). The relations between current and arc length at constant voltage and between current and voltage at constant arc length give rise in each case to simple families of curves. It is convenient to think of the relation between current, voltage and arc length as a three dimensional surface. A plane, $I=$constant, $V=$constant or $L=$constant, intersects this surface on one of the characteristic curves of the arc.

The cadmium photoelectric cell in uviol glass has been used as a means of measuring the ultraviolet output of the arc. This cell, origin-

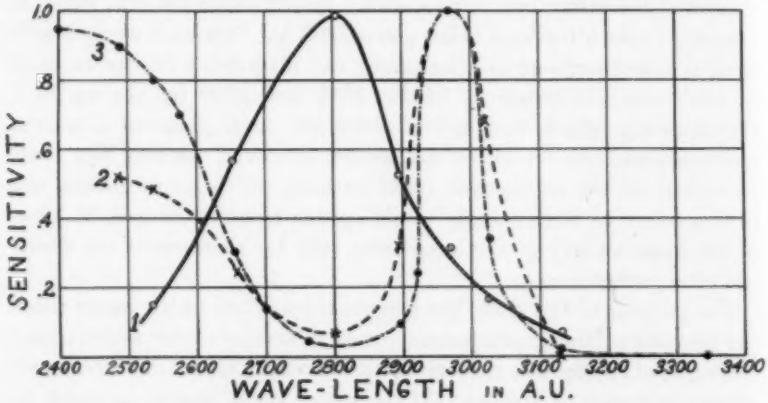

FIG. 1. *Sensitivity curve of the cadmium photoelectric cell in uviol glass (1); of the skin according to Hausser (2), according to Luckiesh, Holladay and Taylor (3).*

ally proposed by Dorno[6] for this purpose, responds to wave lengths between about 3200 and 2500 AU. (curve 1, Fig. 1) which is not greatly different from the sensitivity curve of the skin[7,8] (curves 2 and 3, Fig. 1). Furthermore detailed spectral energy distribution curves which we have made in this laboratory show that the spectral energy

[5] Heubach, Elektro-technische ZS. *13*, 460; 1892.
[6] Dorno, Strahlentherapie, *14*, 25; 1922.
[7] Hausser, Strahlentherapie, *28*, 25; 1928.
[8] Luckiesh, Holladay, Taylor, J.O.S.A., *20*, 423; 1930.

distribution in this region of a therapeutic arc with given carbons has the same relative form regardless of the current and voltage over a reasonable range. Since this is so, any device which responds to this region of the spectrum gives a measure of the relative therapeutic efficiency of different arcs, with the same type of carbon.

Experimental Procedure

The carbons were burned in a simple form of open vertical trim hand-feed arc. The arc length was adjusted by a rack and pinion operated by a hand wheel and was readily set while the arc was burning. Current was drawn from the commercial 110 volt, 60 cycle mains. An adjustable nichrome wire resistance served as ballast. This resistance had sufficient current carrying capacity so that it was not much heated by the current although as a matter of fact the particular value or constancy of the ballast resistance does not affect the conclusions. Ac volt meters read the line voltage and arc voltage and an ac ammeter read the arc current. The photoelectric cell in a suitable box was placed 30 inches from the arc on a line perpendicular to the arc length. The cell voltage was 220 volts. The photoelectric current was read by a galvanometer of 1100 ohms resistance and figure of merit $.8 \times 10^{-9}$ amperes/mm on a scale at about 21 feet from the galvanometer.

In making an observation the ballast resistance was set at some arbitrary value, the arc struck and its length adjusted by the hand feed until the desired arc current was obtained. The arc current and voltage, line voltage, and galvanometer reading were taken. The line was then opened and the arc length measured.

The carbons used do not burn perfectly steadily and fluctuations due to core and shell irregularities in general caused variations of arc current and voltage which in turn caused fluctuations of the ultraviolet output of the arc. These variations made the reading of all instruments difficult. Also to obtain consistent results new carbons must burn long enough to shape themselves to their characteristic rounded tip and for temperature equilibrium to be attained. Then too the length of carbons is important. It was found that with carbons less than a certain length, which depends on the nature and size of the carbons, the loss of heat along the carbons to the metal holders by conduction disturbs the equilibrium with the result that the arc length and the ultraviolet output are greater than with long carbons at the same arc current and voltage. These irregularities are unavoidable and are

characteristic of the arc under actual operating conditions. We attempted to minimize all such irregularities by taking the average of several observations at each point.

The carbons used were National Eveready Therapeutic Carbons of the following types and sizes: Sunshine 10 mm, B 10 mm, C 8, 10 and 13 mm, Supertan 10 mm. A few readings were also made on Eveready 13 mm Projector Carbons and 13 mm Conradty-Noris Vacuum Carbons with neutral core.

Readings were made at 10, 12.5 and 15 amperes on 8 mm carbons, 15, 20 and 25 amperes on the 10 mm, and 25, 30 or 35 on the 13 mm. These are the ranges of current recommended by the makers for these sizes. Since the I vs. V curves at different arc lengths have nearly the form $V=$constant (at a given arc length the voltage is constant), it is possible with a fixed ballast resistance to have any arc current in the working range by varying the arc length. Thus at one setting of the resistance, a point at each of the desired three currents could be obtained.

Readings were made on each type over a wide range of voltage—usually 20–50 volts for the Eveready Therapeutic Carbons and 40–70 volts for the neutral cored carbons.

RESULTS

From the data the characteristic curves for the arc may be plotted for any pair of coordinates V, I and L, as well as curves for the dependence of ultraviolet on these factors.

1. DEPENDENCE OF VOLTAGE ON ARC LENGTH

As mentioned above it has long been recognized that these curves are straight lines of the form $V=V_0+bL$. It was first pointed out by Schwendler[9] in 1879 that the voltage for an arc of a given length was independent of the current. This law is not quite true for solid carbons according to Mrs. Ayrton.[10] That this law as well as the linear relation between voltage and arc length is closely followed by the cored carbons used in this work is shown by Fig. 2. In Fig. 2 are plotted observed arc lengths as a function of voltage for 8 mm Therapeutic C Carbons. These data were taken at 10, 12.5, and 15 amperes. In the figure the 10 ampere values are in circles, the 12.5 in squares and the 15 in triangles. It will be seen that within the accuracy of observation all points

[9] Schwendler, Electrician, *2*, 107 and 117; 1879.
[10] Ayrton, *The Electric Arc* London 1903, 132.

fall on the curves $V=16+16.1\ L$ (L in cm). The other carbons used gave similar results. Observed values of V_0 and b are given in Table 1.

TABLE 1

Carbon Type	$V_0 \pm 0.2$	b	Current Values
10 mm Eveready B	16	16.5	15, 20, 25
8 mm Eveready C	16	16.1	10, 12.5, 15
10 mm Eveready C	16	16.3	15, 20, 25
13 mm Eveready C	16	22.2	25
10 mm Eveready Supertan	16	17.5	15, 20, 25
10 mm Eveready Sunshine	16	18.5	15, 20, 25
13 mm Eveready Projector	27	26.3	35
13 mm Conradty-Noris	42	16.2	35

Heubach[5] for a 4.4 ampere ac arc between neutral cored carbons gave $V=19.8+2.2L$; for a 6.5 ampere arc $V=20.4+1.8L$, while for solid carbons at 5.3 amperes he got $V=44.5+4.4L$, and at 9.2 amperes $V=45.8+3.1L$. It will be seen that Schwendler's law is not followed in either case but that V_0 is independent of current.

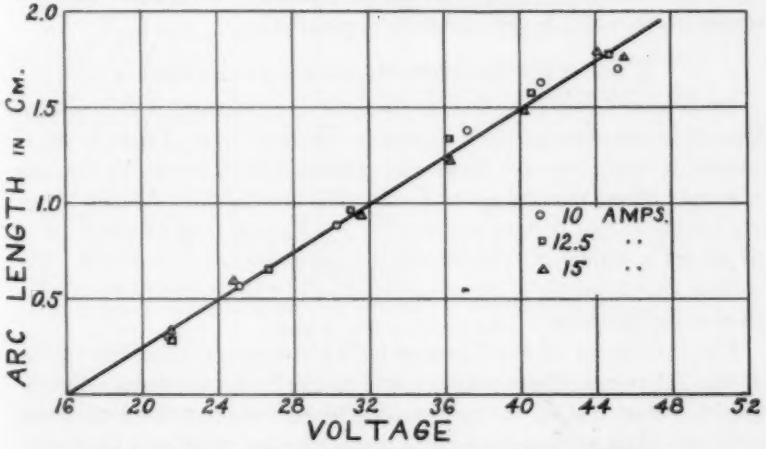

FIG. 2. *Arc length as a function of voltage for 8 mm Eveready Therapeutic C Carbons at 10, 12.5, and 15 amperes.*

It would seem from our results that V_0 is a function chiefly of the nature of the carbon itself, and not of size, core or current, while b depends on the core and in some cases (not however for Eveready carbons) on the current.

As mentioned above the arc length is affected by the mounting of the carbon. Cooling, by conduction or otherwise, causes greater arc lengths at a given voltage. This phenomenon seems to affect, at least for reasonable amounts of cooling, only the values of b in the characteristic equation and not V_0. For example with the 10 mm Supertan Carbons at 25 amperes we have obtained by varying carbon lengths values of b as low as 15.7 and as high as 24.3, while V_0 remains 16 volts within the accuracy of our observations. With lower current values b fluctuated much less and the value given of 17.5 gives the arc voltage under our normal condition of operation (carbons about 6–7 cm long) within about 5%. For the smaller carbons at correspondingly lower current values the fluctuations are even less as may be seen from Fig. 2. The arc length is also dependent on the orientation of the arc, being a maximum for a vertical trim and a minimum for a horizontal trim.

These variations may of course be thought of in another way. Instead of fluctuations in arc length at a given voltage, one may say that the voltage required to maintain an arc at a fixed length depends on the heat conduction or ventilation or orientation of the arc. Stated thus the phenomenon is well known for the ordinary therapeutic mercury arc in quartz which is very sensitive to ventilation.

2. RELATION BETWEEN VOLTAGE AND CURRENT

From what has been said under the preceding heading it is clear that the other two characteristic curves of the arc, V vs. I and L vs. I, consist in each case for Eveready Therapeutic Carbons of families of curves (lines) parallel to the I axis. Thus in the V vs. I plane for an arc length L, the voltage is given by $V=V_0+bL$, and in the L vs. I plane, for a voltage V_1, the arc length is given by $L=(V_1-V_0)/b$. The surface characteristic in the range studied of the Eveready carbons is thus an inclined plane.

Fig. 3 shows for 10 mm Therapeutic B Carbons several of the curves in the V-I plane. From such a curve as this, one may deduce the required ballast and operating behavior of an arc operated with these carbons. Thus suppose we wish to operate an arc, resistance ballasted, on a 100 volt line with a maximum current never to exceed 31 amperes and a striking mechanism which opens the arc initially to .3 cm. A line, AB, intersecting the V axis at 100 volts and the I axis at 31 amperes will cut the arc voltage lines at the current which the arc will draw at the arc length corresponding to any line. The arc will thus begin to burn at 24.7 amperes and the current will decrease, as the arc

length increases by carbon consumption. The arc voltage curves do not extend as straight lines to the V axis but curve up as I decreases. Eventually the line AB will fail to intersect a voltage line—the arc is

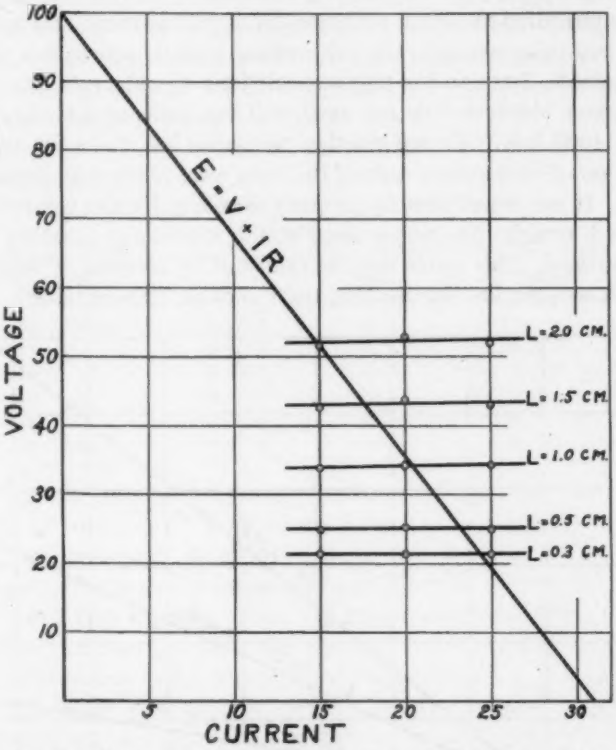

Fig. 3. *Voltage as a function of current for 10 mm Eveready Therapeutic B Carbons at several arc lengths.*

unstable and goes out. These unstable parts of the voltage curves have not been determined in this work.

3. Relation between voltage and ultraviolet output

Griffith, Taylor and Jack[2] have shown that in the biologically important wave-length region the radiation from the arc poles is still appreciable. In our results the total radiation from all parts of the arc falls on the photoelectric cell. Thorsen[3] studied the dependence upon voltage of ultraviolet output for the arc flame only which he focused

on the slit of a spectrograph. Thorsen found for the cyanogen bands that the ultraviolet radiation follows the law $U=k\sqrt{V-V^0}$ where U is ultraviolet intensity, k is a constant, V the arc voltage and V^0 a constant—a law which was first announced by Hallwachs.[11] Hallwachs measured the total radiation from neutral cored and solid carbons with zinc photoelectric cells whose spectral sensitivity was not determined. Data on the photoelectric long wave-length limit of zinc by various observers[12] do not agree well but indicate that the limit is below 4000 A.U. To see whether the same law holds for the vital radiation of therapeutic carbon the data were plotted on logarithmic paper. It was found that by properly choosing V^0, the observed data lay on a straight line whose slope was 2, within the accuracy of our observations. This result may be exhibited by plotting U^2 against V, when a straight line intersecting the V axis at V^0 is obtained. Fig. 4

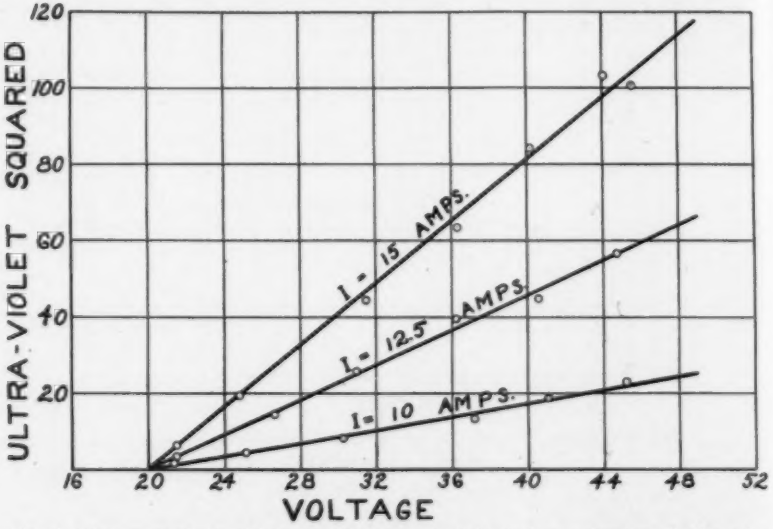

FIG. 4. *The square of ultraviolet energy (galvanometer deflection) as a function of voltage for 8 mm Eveready Therapeutic C Carbons at 10, 12.5, and 15 amperes.*

shows such a plot for 8 mm. Therapeutic C Carbons at 10, 12.5 and 15 amperes. It will be seen that in each case the result is a straight line with $V^0=20$. Thorsen found for neutral cored carbons that the

[11] Hallwachs, Ann d. Phy., *13*, 38; 1904.
[12] Gudden, *Lichelektrische Erscheinungen* Berlin 1928, 40.

law was followed in the range 48–60 volts with $V^0=43$ volts, while below 48 volts, a linear relation $U=K(V-33.1)$ held. No departure from linearity is apparent in our data. However, since with therapeutic carbons arcs may be operated down to 16 volts (V_0) it is apparent that at low voltages a different law must hold. At these low voltages the arcs are unsteady and the ultraviolet emission small, so that results are difficult to obtain, hence we have not felt it to be of practical importance to attempt to study this region.

For the various carbons used we have determined the values of V^0 shown in Table 2.

TABLE 2

Carbon Type	V^0	Size	Current
Eveready Sunshine	16±0.5	10	15
Eveready B	22± 1	10	15, 20, 25
Eveready C	20±0.2	8, 10, 13	10, 12.5, 15, 20, 25
Supertan	20±0.5	10	15, 20, 25
Projector	40± 1	13	35
Conradty-Noris	41± 1	13	35

The scale of ultraviolet intensity in Fig. 3 and the value of k in the equation $U=K\sqrt{V-V^0}$ is of course arbitrary and depends on the cell and galvanometer sensitivities and other factors. It has little meaning to give numerical values of k except for different current values for the same type carbons. Since the photoelectric cell is a selective detector of radiation and since different carbons have different spectral energy distribution in the region of the cell's sensitivity, the galvanometer deflections with different carbons do not compare the relative biological or erythema-producing efficiency of the different carbons. In general it may be said that for Eveready Therapeutic Carbons for all of which V^0 is about 20 volts, k has about the same value for B, C, and Supertan Carbons while the response of the photoelectric cell to Sunshine Carbons is about $\frac{1}{4}$ as much for the same operating conditions.

4. VARIATION OF ULTRAVIOLET OUTPUT WITH CURRENT

For several of the carbons data were obtained at three current values so that the variation of ultraviolet with current could be studied. It has seemed best to take values from the U^2 vs. V lines at a given voltage, take their square root and use these roots as values of ultraviolet. This eliminates individual fluctuations in measurement of output at

a given voltage and current and is preferable to taking values from the U vs. V curves which are not so precisely determined. That is, the U^2 vs. V straight line is more definitely determined by say seven points of our accuracy of determination than is the U vs. V parabola.

Thorsen[3] found the intensity of cyanogen bands to be proportional to the square of the current. However, as noted in Section 3 he focused an image of the arc flame on the slit of a spectrograph and so really studied the intrinsic brilliancy of the flame. It may be expected that the total radiation of the whole arc and poles may not follow the same

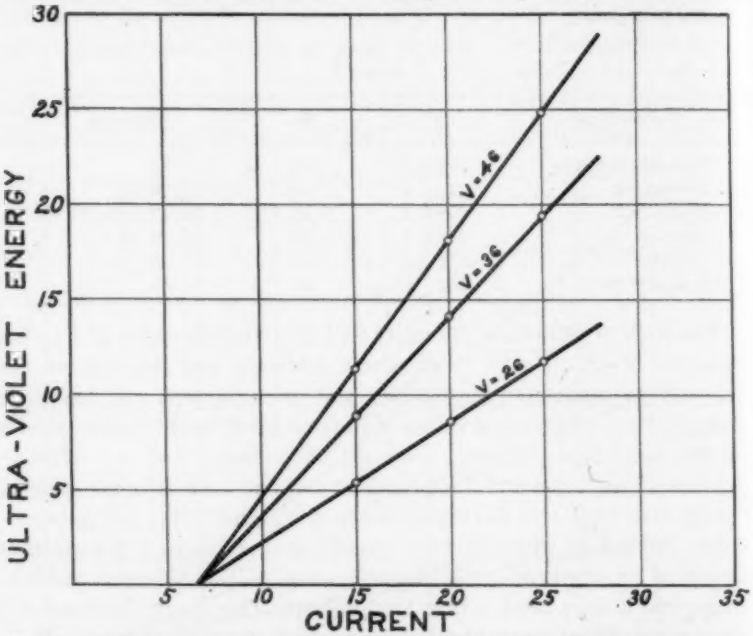

FIG. 5. *Ultraviolet energy (galvanometer deflection) as a function of current for 10 mm Eveready Superian Carbons at 26, 36, and 46 volts.*

law of current dependence even though the laws of voltage variation are the same. In fact we find that our observations do not at all fit a current square law. Taking pairs of values from the data as described in the preceding paragraph and assuming that $U_1/U_2=(I_1/I_2)^n$ the values of n so calculated range from 1.4 to 2.2 and in no case are the three values obtained from the three possible pairs of observations on any carbon in reasonably close agreement.

Hallwachs[11] found that his zinc photoelectric cell data, referred to

above, obeyed the law, $U=C(I-I_0)$. A plot of ultraviolet intensity against current showed at once that this was also the case for our data for the ultraviolet radiation recorded by a cadmium cell. In Figure 5 is shown such a plot for 10 mm, Supertan Carbons. It will be seen that the three points fall on a line. I_0 is seen to be 6.5 for these carbons. For 8 mm Therapeutic C Carbons I_0 was found to be 5.6, and for 10 mm Therapeutic B Carbons, 8.0 amperes. The slope of the I vs. U curve is of course an arbitrary function of the cell and galvanometer used. It is interesting to note that I_0 is a function of both carbon size and type in contrast to V^0 and V_0 which for Eveready carbons were found (Table 2) to be nearly independent of size and type of carbon.

5. General Expression for Ultraviolet Output

The results of sections 4 and 5 may be combined into a general expression, $U=a(I-I_0)\sqrt{V-V^0}$ first given by Hallwachs.[12] The output of an ac therapeutic carbon arc thus depends on three constants—I_0 and V^0 which are constants of the carbons alone, and a which depends not only on the nature of the carbons but also on their orientation and mounting and on the units in which U is to be expressed. Thus if a spectroradiometric analysis of the arc in absolute units at some definite current and voltage had been made, a might be chosen to give the radiant energy in any chosen band of wavelengths falling on a surface at a given distance from the arc in ergs/sec/mm.^{2} Or again to measure the erythema-producing efficiency of the arc with particular carbons, a might be so chosen that U would be the reciprocal of the time in minutes required for the arc to produce an average erythema reaction on normal adult skin. The product of U by the time of treatment with the arc would then give the dose in Pohle's ultraviolet units.[13] Of course for commercial lamps with reflectors, a includes the contribution of the reflector. The reflector factor varies with design and efficiency of the reflector and with the beam spread of the reflector. Thus for various commercial lamps tested the axial intensity of the beam is increased by the reflector to from two to six times the intensity of the beam from the bare arc.

6. Variation of Arc Output with Time

It may be seen from the characteristic curves that if an arc be operated at a constant arc length it will also operate at a constant arc voltage and the current through it may be varied only by changing

[13] Pohle, Am. J. Roentgenol and Rad. Ther., *15*, 193; 1926.

the ballast resistance at a constant line voltage, or the line voltage with a fixed ballast. Such an arc will show an output depending on the current $[C(I-I_0)]$ and under fixed conditions of line voltage and ballast will have a constant output.

On the other hand if an arc be operated at constant line voltage and ballast, being struck and allowed to burn without further adjusting, the current and voltage are uniquely determined by the arc length. As explained in Section 2, and Fig. 3, any current may be had only at one arc length and voltage which is determined by the intersection of the line, $E_{\text{line}}=V_{\text{arc}}+IR_{\text{ballast}}$ with one of the family of arc characteristics $V_L=$constant (at constant arc length). Hence $V_{\text{arc}}=E-IR$

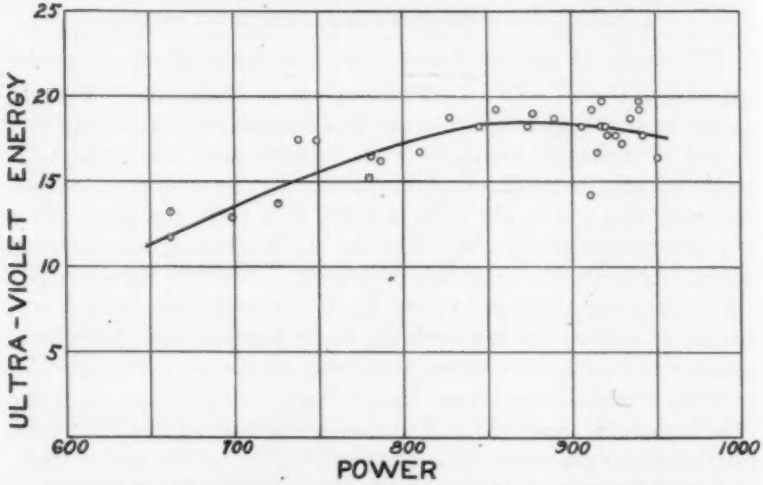

FIG. 6. *Ultraviolet energy (galvanometer deflection) as a function of power consumed in the arc for a resistance ballasted arc with 10 mm Eveready Therapeutic B Carbons in a time test.*

and power in the arc $=W=VI=EI-I^2R$. This is a parabola with maximum W for $I=E/2R$ and $V=E/2$. In actual practice the voltage is usually near this point and the arc operates over a large part of its run at fairly constant power. The ultraviolet output of course is not exactly proportional to power but as shown in section 5, is governed by $U=a(I-I_0)\sqrt{V-V^0}$. However, the ultraviolet output actually is often surprisingly constant. Fig. 6 shows a plot of the ultraviolet output (galvanometer deflection) against power for a simple resistance ballasted arc with 10 mm B Carbons. The line voltage was 100 volts, the arc drew on short circuit 31 amperes, it was struck at 25 amperes

and 26 volts and burned 10 minutes, the current gradually falling to 18 amperes and the voltage rising to 52 volts. The points are nearly in chronological order from left to right and it will be seen that except for the first four points taken in the first 90 seconds the output is essentially constant. Various commercial lamps tested show similar characteristics, showing greater or less variation in power or ultraviolet output. Core irregularities and instantaneous fluctuations in current give rise to at least as large variations.

Summary

It has been shown that in the practical working range of current and voltage the operating characteristics of a therapeutic carbon arc are described by the equation:

$$V = V_0 + bL \qquad \text{(at all currents)}$$

where V = arc voltage, L = arc length, V_0 is a constant depending on the type of carbons used, and b is a constant depending not only on the carbon type but on the mounting and the arc orientation. The method of determining the arc current for any given values of line voltage and ballast resistance is discussed.

The ultraviolet output of a therapeutic arc in the region of biological importance is shown to be given by:

$$U = a(I - I_0)\sqrt{V - V^0}$$

where U = ultraviolet intensity, I = arc current, V = arc voltage, I_0 and V^0 are constants depending on the carbon type, and a is a constant depending not only on carbon type but also on mounting, arc orientation, and the units in which U is expressed.

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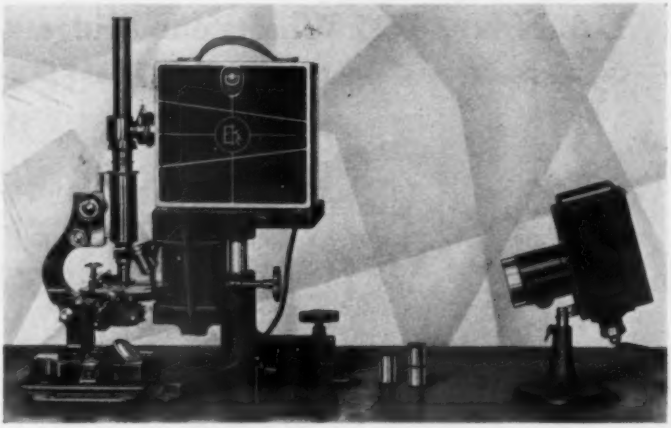

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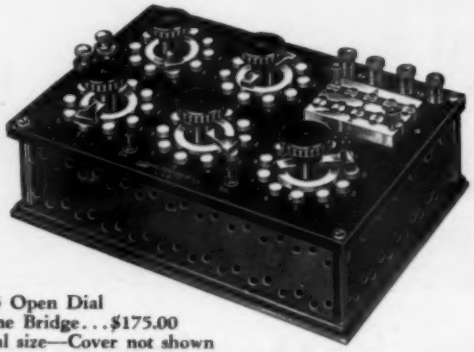

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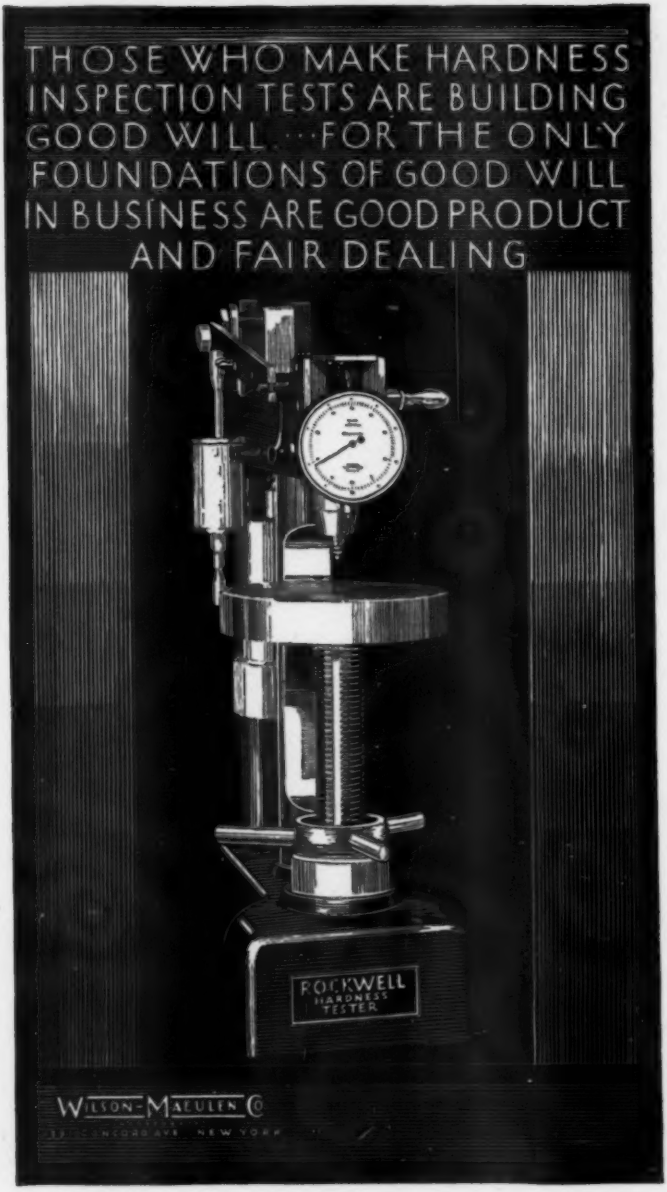
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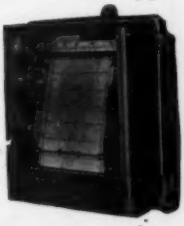

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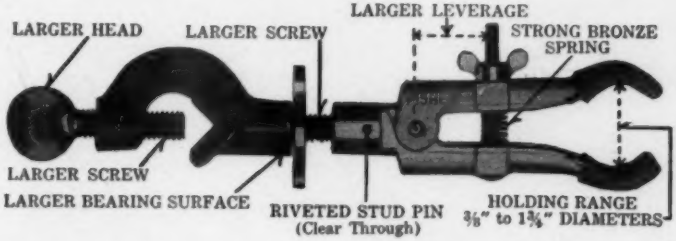

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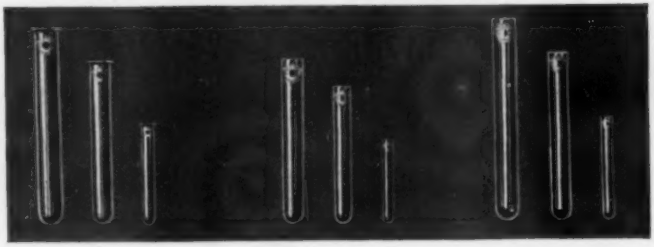

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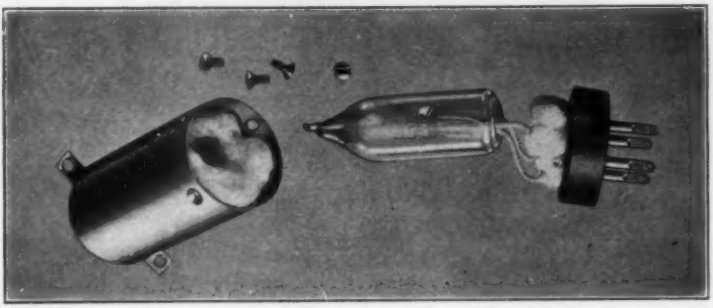

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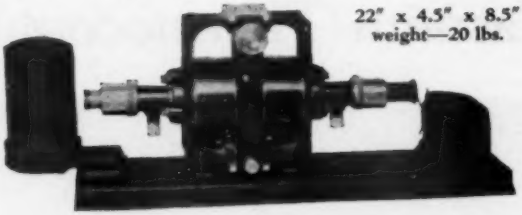

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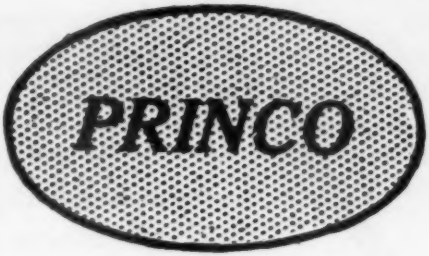

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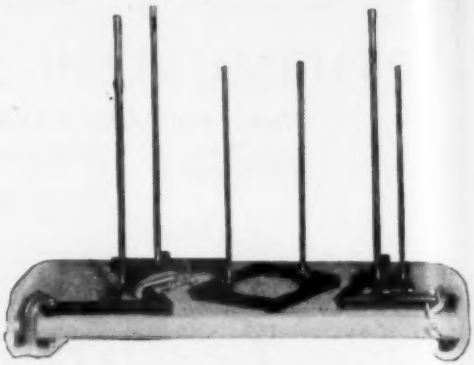

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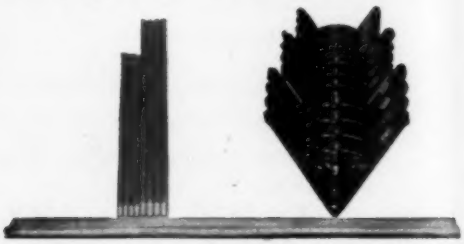

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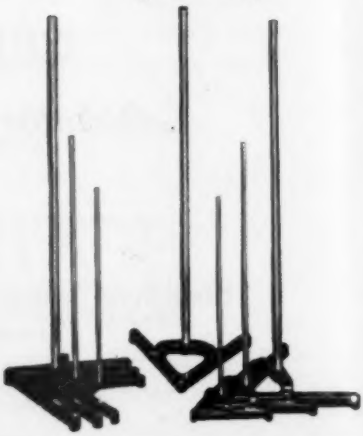

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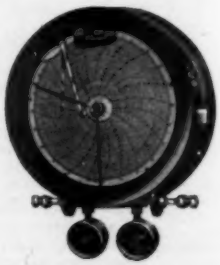

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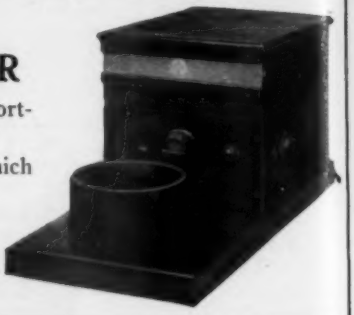

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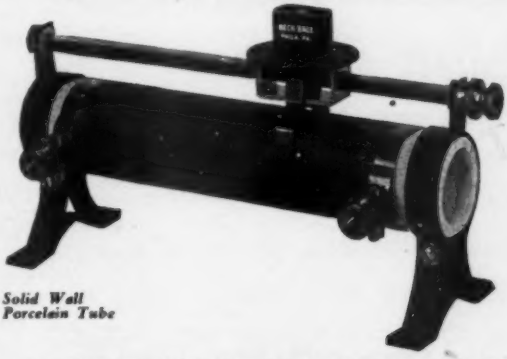

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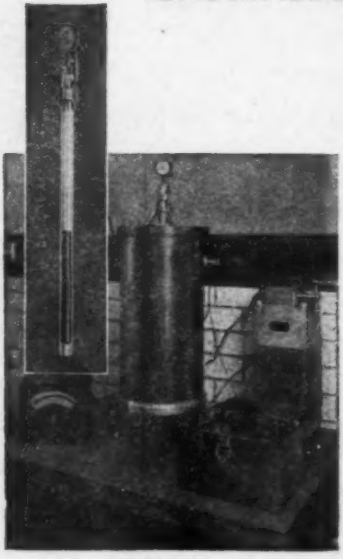

Fig. 2

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